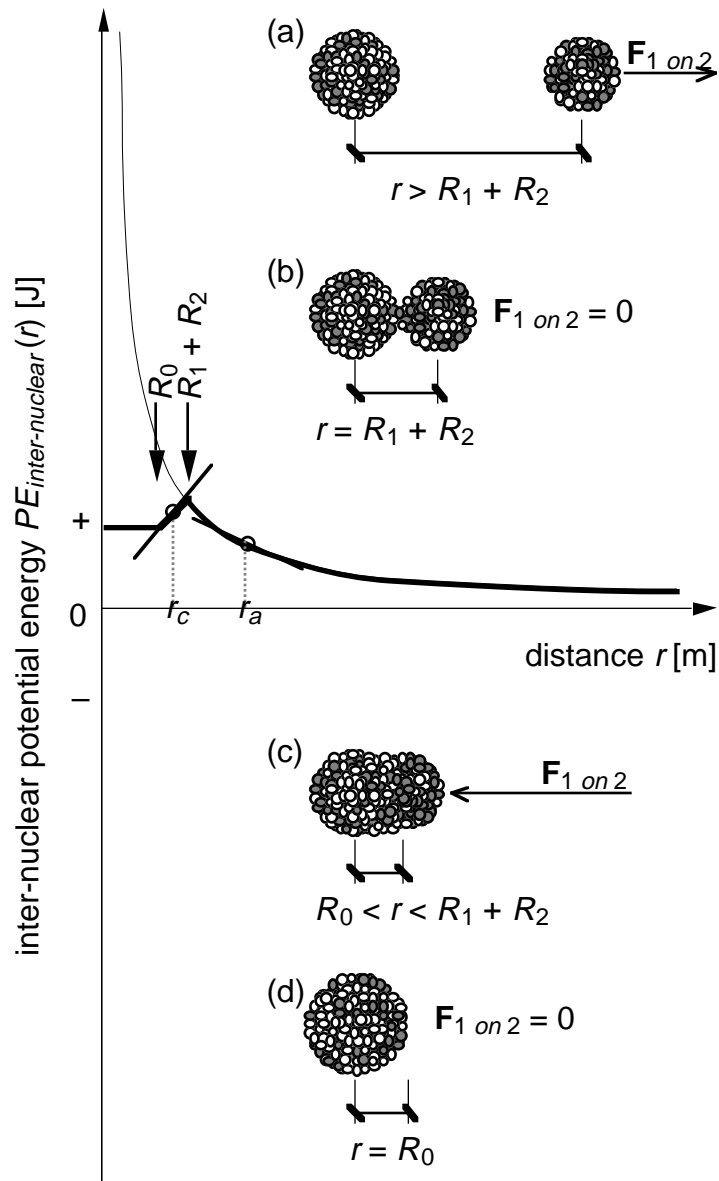


Physics 7C

Student Packet



Winter Quarter 2003
University of California, Davis

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What's on the cover?

Inter-nuclear potential energy is stored in the position of a test nucleus, relative to a source nucleus, and is the result of **electric potential energy** and **nuclear strong potential energy**. (In Physics 7A we considered the "Lennard-Jones potential," which was actually the **inter-atomic potential energy** of the relative positions of two atoms, when their nuclei are very distant from each other.)

A **gradient relation** can be used to relate forces and PE graphs. The *magnitudes* of the forces exerted between these interacting atoms can be calculated from the slopes of the $PE_{\text{inter-nuclear}}$ graph; the *directions* of the forces involved must point towards decreasing potential energy.

The process shown on the cover page is for endothermic **fusion** (which is the reverse of exothermic **fission**).

When the nuclei are not in contact with each other (as shown in (a)), the only contribution to inter-nuclear potential energy is the electric potential energy of the nuclei exerting repulsive forces on each other, and has a $1/r$ dependence. Bringing the nuclei closer together (as shown in (a)→(b)) results in increasing their electric potential energy, and thus increases the inter-nuclear potential energy of this two-nucleus system.

However, if these nuclei are brought close enough to each other, then strong bonds can form between nucleons that touch, releasing energy to the environment. Attractive nuclear strong forces are exerted on the two nuclei (as shown in (c)). This reduces the nuclear strong potential energy (even though electric potential energy is *still* increasing), and thus overall reduces the inter-nuclear potential energy of this two-nucleus system, until the single resulting nucleus is formed (as shown in (d)).

(*See fission, fusion, gradient relation, potential energy (electric), potential energy (inter-atomic), potential energy (inter-nuclear), potential energy (nuclear strong).*)

Dedicated to my wonderful wife, H. M.

A "Waifer X® Industries, Inc. Book"™

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Physics 7C Student Packet

Winter Quarter, 2003

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Constants and Conversion Factors

Constants

G	gravitational constant	$6.67 \times 10^{-11} \text{ N} \cdot \text{m}^2 / \text{kg}^2$
k	electric force constant	$8.99 \times 10^9 \text{ N} \cdot \text{m}^2 / \text{Coul}^2$
μ_0	magnetic permeability	$1.26 \times 10^{-6} \text{ Tesla} \cdot \text{m} / \text{Amps}$
c	velocity of light	$3.00 \times 10^8 \text{ m/s}$
m_e	mass of the electron	$9.11 \times 10^{-31} \text{ kg} = 0.0005486 \text{ u}$
m_p	mass of the proton	$1.6726 \times 10^{-27} \text{ kg} = 1.00728 \text{ u}$
m_n	mass of the neutron	$1.6750 \times 10^{-27} \text{ kg} = 1.00866 \text{ u}$
e	fundamental charge	$1.602 \times 10^{-19} \text{ Coul}$
h	Planck's constant	$6.626 \times 10^{-34} \text{ J} \cdot \text{s}$
k_B	Boltzmann's constant	$1.38 \times 10^{-23} \text{ J/K}$

Terrestrial constants

g_E	gravitational field magnitude (at or near Earth's surface)	9.8 N / kg
M_E	mass of the Earth	$5.98 \times 10^{24} \text{ kg}$
R_E	radius of the Earth	$6.37 \times 10^6 \text{ m}$

Conversion factors

Energy

1 eV	$= 1.602 \times 10^{-19} \text{ J}$
1 MeV	$= 1 \times 10^6 \text{ eV}$
1 "kiloton" of TNT	$= 4.184 \times 10^{12} \text{ J}$
1 cal	$= 4.184 \text{ J}$
1 kcal ("food Calorie")	$= 4,184 \text{ J}$

Length

1 m	$= 39.37 \text{ in} = 3.281 \text{ ft}$
1 km	$= 0.6214 \text{ mile}$
1 Å (angstrom)	$= 10^{-10} \text{ m}$
1 nm	$= 10^{-9} \text{ m}$
1 fm	$= 10^{-15} \text{ m}$

Speed

1 m/s	$= 3.28 \text{ fps (ft/s)} = 2.24 \text{ mph (miles/hr)} = 3.60 \text{ kph (km/hr)}$
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Force

1 N	$= 0.225 \text{ lb}$
-----	----------------------

Mass

1 kg	$= 0.0685 \text{ slugs}$
1 u	$= 1.66054 \times 10^{-27} \text{ kg} = 931.5 \text{ MeV}/c^2 = 1.49242 \times 10^{-10} \frac{\text{J}}{c^2}$

Academic Calendar

(Periodically updated with announcements at <http://physics7.ucdavis.edu/>)

Monday	Tuesday	Wednesday	Thursday	Friday
Jan 6 <i>Lecture 11.1</i>	<u>7</u> <i>DLM 01</i>	<u>8</u> <i>DLM 01</i>	<u>9</u> <i>DLM 01 DLM 02</i>	<u>10</u> <i>DLM 02</i>
13 <i>Lecture 12.1</i>	<u>14</u> <i>DLM 02 DLM 03</i>	<u>15</u> <i>DLM 03</i>	<u>16</u> <i>DLM 03 DLM 04</i>	<u>17</u> <i>DLM 04</i>
20 <i>MLK, Jr. holiday</i>	<u>21</u> <i>DLM 04 DLM 05</i>	<u>22</u> <i>DLM 05</i>	<u>23</u> <i>DLM 05 (No new DL)</i>	<u>24</u> <i>Lecture 12.2 Quiz 11 (No DL)</i>
27 <i>Lecture 13.1</i>	<u>28</u> <i>DLM 06</i>	<u>29</u> <i>DLM 06</i>	<u>30</u> <i>DLM 07</i>	<u>31</u> <i>DLM 07</i>
Feb 3 <i>Lecture 13.2</i>	<u>4</u> <i>DLM 08</i>	<u>5</u> <i>DLM 08</i>	<u>6</u> <i>DLM 09</i>	<u>7</u> <i>DLM 09</i>
10 <i>Lecture 14.1 Quiz 12</i>	<u>11</u> <i>DLM 10</i>	<u>12</u> <i>DLM 10</i>	<u>13</u> <i>DLM 11</i>	<u>14</u> <i>DLM 11</i>
17 <i>President's Day holiday</i>	<u>18</u> <i>DLM 12</i>	<u>19</u> <i>DLM 12</i>	<u>20</u> <i>DLM 13</i>	<u>21</u> <i>DLM 13</i>
24 <i>Lecture 14.2 Quiz 13</i>	<u>25</u> <i>DLM 14</i>	<u>26</u> <i>DLM 14</i>	<u>27</u> <i>DLM 15</i>	<u>28</u> <i>DLM 15</i>
Mar 3 <i>Lecture 15.1</i>	<u>4</u> <i>DLM 16</i>	<u>5</u> <i>DLM 16</i>	<u>6</u> <i>DLM 17</i>	<u>7</u> <i>DLM 17</i>
10 <i>Lecture 15.2 Quiz 14</i>	<u>11</u> <i>DLM 18</i>	<u>12</u> <i>DLM 18</i>	<u>13</u> <i>DLM 19</i>	<u>14</u> <i>DLM 19</i>
17 <i>Final Exam 8:00–10:00 am</i>	<u>18</u>	<u>19</u>	<u>20</u>	<u>21</u>

There are two DL meetings a week, TuWTh and ThFTu

Note that Friday, 1/24 is an Academic Monday, and Quiz 11 will be given in lecture that day

Except for Quiz 11, all subsequent quizzes are on Mondays, so schedule your office hour visits accordingly ahead of time

There are only nine lectures in Winter quarter 2003!

The Final Exam is on a Monday, so schedule your office hours visits accordingly ahead of time

Course Policy

Welcome to Physics 7C!

Physics 7C is the last quarter in a three-quarter course designed in such a way to maximize student learning, incorporating the results of current physics education research. After two quarters of Physics 7A and 7B, you are well aware of how this course is structured quite differently from most other introductory science classes, either at UC-Davis or elsewhere. The time spent in lectures has been significantly reduced; and is not meant to be the primary means of acquiring and familiarizing yourself with course content. Lectures are intended to provide you with an overview and framework to guide you in your primary exposure to the course content in the five hours per week of discussion/labs (DLs), as well as the significant time spend outside of class doing exit handout homework.

Contact Information

(While Physics 7C is normally team-taught by several instructors, due to low enrollment there is only a single instructor during this quarter.) When using e-mail, include "2003-01 Phy 7C" in the subject line. If you are browsing from a public computer terminal, make sure to *include* your return address, as it will not show up in the e-mail header. *For obvious reasons your e-mail cannot be replied to unless you have explicitly included your return address in the header and/or body of your message.*

Instructor contact
information

Dr. Patrick M. Len, Terminated Sixth-Year Lecturer

office: 112 Walker Annex¹

phone: (530)-754-8697²

e-mail: pml@waiferx.com

office hours: W 9:00-10:00 am, Walker Annex 113, or by appointment
(I have an open-door policy during most mornings. You are certainly welcome to drop in with short questions at any time; but if it is important for me to be in and to meet you at a specific time, feel free to make an appointment.)



Lectures are M 9:00-10:20 am in 66 Roessler. It is important that you attend every lecture. The lectures are designed to organize and facilitate your learning process in DL.

¹ Dr. Len will not be offered a sixth-year "eye-of-the-needle" review by the College of Letters and Sciences, and thus will be precluded from teaching at any and all University of California campuses after the end of Winter 2003. You should make sure to conclude all Physics 7C-related academic contact with Dr. Len *before* March 21, 2003, as he will no longer be allowed to perform academic duties nor maintain an official presence on campus after such time.

² After March 21, 2003, all correspondence must be sent via e-mail to pml@waiferx.com (this address is permanent). No messages can or will be forwarded to Dr. Len from this campus phone number after such time.

Discussion/Labs

There are seven different DL sections of no more than 25 students each. Each DL meeting lasts two hours and 20 minutes, and there are two meetings per week (TuWTh and ThFTu), in TB 114 (which is the yellow temporary building, across California Avenue from the east entrance of Haring Hall).

Enrollment for Physics 7C is handled solely through the DL teaching assistants

During much of a DL meeting you will be working in a small group of five students. In your group you will design and perform experiments to explore physics concepts, discuss the results of these experiments with the class as a whole, and work within your small group and the whole class to gain and understanding of how these concepts can be applied to a wide range of real-world problems and situations. At the end of each DL you will pick up an exit handout containing FNT ("for next time") mandatory homework assignments, which will help you consolidate what you have learned in that DL, and to help you prepare for the next DL.

You are expected to participate in both your small group and whole class DL discussions, activities, and experiments, and to have completed any and all required exit handout assignments. Attendance in DL is mandatory (see "Discussion/Laboratory Grading" below).

Discussion/Lab Teaching Assistants

All DL instructors hold weekly office hours, which are open to *all* students in *all* DL sections. When using e-mail, include "2003-01 Phy 7C" in the subject line. If you are browsing from a public computer terminal, make sure to *include* your return address, as it will not show up in the e-mail header. *For obvious reasons your e-mail cannot be replied to unless you have explicitly included your return address in the header and/or body of your message.*

Teaching assistant contact information

Cary Allen, Teaching Assistant

office: 104 Phys/Geo
 e-mail: allen@physics.ucdavis.edu
 office hours: _____



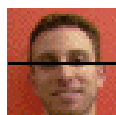
Brooke Haag, Teaching Assistant

office: 331 Phys/Geo
 e-mail: angellight@aol.com
 office hours: _____



David Michaels, Teaching Assistant

office: 104 Phys/Geo
 e-mail: michaels@physics.ucdavis.edu
 office hours: _____



Randy Nelson, Teaching Assistant

office: 113/434 Phys/Geo

e-mail: nelson@physics.ucdavis.edu

office hours: _____



The *Physics 7C Student Packet* is mandatory reading for the material covered in Physics 7C in Winter quarter 2003

The *Physics 7C Course Notes* is to be considered optional and adjunct to the material covered in Physics 7C in Winter quarter 2003

Learning Resources

- The Physics 7C webpage may be found online at <http://physics7.ucdavis.edu/>, where all downloadable *.pdf materials will be posted.
- *Physics 7C Student Packet, Winter Quarter 2003* is available at Navin's Copy Shop (231 Third Street; 758-2311). *Note that this packet contains information specific only to this course as taught in Winter quarter, 2003, and cannot and should not be used in Physics 7C as taught in future quarters.*
- *Physics 7C Course Notes* (W. H. Potter, 2002, J. Wiley Custom Services, ISBN 0-471-23044-8) is also available at Navin's Copy Shop (231 Third Street; 758-2311). *This is to be considered optional and adjunct to the material covered in Physics 7C, Winter quarter 2003.*
- Exit handouts and supplementary DL materials will be distributed in a downloadable *.pdf format from the Physics 7C webpage.
- Course binder—you are responsible for organizing the sizable amount of notes and DL materials in the manner that is most efficient for your studying purposes.

Course Expectations and Grading

The responsibility for learning is yours. If you participate fully in lecture and in DL, and complete the assigned work, you should do well. It is certainly possible that all Physics 7 students can attain the expected level of understanding. You will not succeed if you do not attend the lectures and the DLs, and/or do not regularly put forth a good effort into the learning activities and assignments.

At left is the current grade distribution for all students on record that have taken Physics 7ABC courses from this instructor (Dr. Len) over the past six years, and is publicly accessible (as well as with all UC-Davis courses) online at <http://www.pickaprof.com/>. *This diagram is included here for comparative and disclosure purposes only, and should in no way be taken as an indication of your possible future performance in this course.*

Your course letter grade in Physics 7C is made up of two components. The first of these is your lecture grade determined by your performance on the four quizzes, and on the Final Exam. The second reflects your participation in and preparation for DL, which modifies your lecture grade.

Your course letter grade will be determined by either of the two weighting schemes listed below, whichever is to your best advantage. *These Winter quarter 2003 weighting schemes are unique and are drastically*

Undergrad - Len

686 students in 74 classes

A: 20.26%

B: 54.08%

C: 24.48%

D: 0.43%

F: 0.72%

Avg GPA: 2.92

different than those given in Physics 7C by other instructors, and these weighting schemes will not necessarily carry over into Physics 7C in a later quarter.

- 60% average of four quizzes + 40% Final Exam ± (DL grade adjustment)
- 20% average of four quizzes + 80% Final Exam ± (DL grade adjustment)

Weighting schemes to determine your course letter grade, whichever is to your best advantage

The maximum possible course grade points is always 4.500. The scale for determining letter grades from grade points is given below. *There is no curve for this class*; your performance is strictly determined relative to this scale.

Your course letter grade is determined by a scale, not by a curve

4.167-4.500	A+
3.834-4.166	A
3.500-3.833	A-
3.167-3.499	B+
2.834-3.166	B
2.500-2.833	B-
2.167-2.499	C+
1.834-2.166	C
1.500-1.833	C-
1.167-1.499	D+
0.834-1.166	D
0.500-0.833	D-
0.000-0.499	F
0.000	Y (Course grade will be determined at a later date by Student Judicial Affairs.)

Physics 7 grading scale, from 0.000 to a maximum of 4.500 points

Quizzes

There will be four quizzes in lecture. Note that quizzes are numbered corresponding to their Blocks (Quiz 11, Quiz 12, Quiz 13, and Quiz 14; note that "Quiz 15" is incorporated into the Final Exam).

There are four quizzes, one for each of Blocks 11-14

Quizzes are closed-book and closed-notes.

Each quiz is 35 minutes long, given in the first 35 minutes of lecture (9:00-9:35 am). If you show up late to lecture on a quiz day, you will only have the remaining time until 9:35 am to take the quiz. It is expected that each and every student makes the necessary arrangements and contingency plans in advance to arrive promptly to lecture on a quiz day.

Quiz tardiness policy

Calculators are allowed for the quizzes; make sure you have a scientific calculator available. The sharing of calculators between students during a quiz is strictly prohibited, and will be considered an act of academic misconduct to be referred to Student Judicial Affairs. The procurement and possession of a scientific calculator for a quiz or the Final Exam is solely the responsibility of each and every student. Under no circumstances is the instructor obliged to provide "loaner" scientific calculators to those students

Scientific calculator policy

There are no make-up quizzes, but a missed quiz score may be "replaced"

Replacement quiz score policy

who fail to bring one. In any case all Physics 7 students are expected to be able to perform any and all calculations by hand; use of scientific calculators on a quiz or the Final Exam should be considered a privilege and not a right.

There will be no make-up quizzes. If for any reason you miss a quiz, please carefully read the following:

- There is no obligation on the part of the instructor to "replace" a missing quiz grade with a score other than zero.
- Under circumstances entirely at the sole discretion of the instructor³, a missing quiz grade may be "replaced" in accordance with your average performance, as compared relative to the class mean on the missing quiz. This is not the same as the simple arithmetic average of your three remaining quiz grades, and as such, the "replacement" quiz grade can and will be significantly lower or higher than the simple arithmetic average of your three remaining quiz grades. This is done in order to normalize your "replacement" quiz score relative to a statistically low or high class mean on a missing quiz. (For example: if you consistently score below the class average on your remaining quizzes, your calculated replacement score will be proportionally below the class average on your excused quiz, and *vice versa*.)
- It is your option to receive a zero for your missing quiz grade if the method described above in determining your "replacement" quiz grade is unacceptable to you.
- You cannot have the score for a quiz that you have already taken *retroactively* replaced, removed, or excused. By showing up to lecture and taking a quiz, your actions imply that you are prepared and able to perform to the best of your ability on the quiz at the assigned date and time, and demonstrate your intent to have your work graded equitably with the rest of the class.

The quizzes (and the Final Exam) will be graded by assigning a rubric code that characterizes your response. These letter codes will be translated into a letter grade on a 4.500 point scale. A description of the rubric codes will be available following each quiz.

If you believe a mistake was made in characterizing your response, you should first check your quiz with the rubric code. Remember, the rubric code is an indication of what you actually wrote down on paper during the quiz. The rubric is not an indication of what you might have been thinking (but did not write down), nor does it ignore material you wrote down but changed your mind about later.

³ These must be legitimate, verifiable circumstances that were *beyond* your control. Complete, relevant documentation *must* be provided by the student, subject to review by the instructor.

Once you are certain a mistake was made, you may request a reconsideration of the coding of a particular response. In order to do this, first carefully read the following:

- Attach to the original quiz a clear written explanation of what you believe your rubric category should be, and why the rubric assigned to your answer was not appropriate. We will only consider changing your rubric category to the new category stated in your petition, so make it clear and concise!
- Regrade submissions without clear and thorough explanations attached will not be re-examined.
- *Do not write anything on the quiz itself!* Any regrade submissions with alterations (whether intentional or accidental) on the quiz itself are required to be referred to Student Judicial Affairs, and a "Y" hold will be issued on your course grade. *Many previous students have been unaware or undeterred by the fact that all regrade submissions are checked for alterations to the original quiz, and have had to be referred to Student Judicial Affairs.*
- Give your regrade request and quiz to the instructor in lecture; all regrade requests must be submitted before taking the next quiz. No quiz regrades will be accepted after the next quiz has been taken. All late quiz regrade requests are categorically refused and returned to the students unreviewed and ungranted.

Quiz regrading policy

Quizzes altered in any manner whatsoever constitutes a case of alleged academic misconduct that must be reported to Student Judicial Affairs

Absolutely no late quiz regrade requests will be accepted

Final Exam

The comprehensive Final Exam will be held from 8:00 - 10:00 am on Monday, 17 March 2003, and will be 120 minutes long. You must take the Final Exam at this assigned time and day. *There will be no make-up Final Exam.* If you cannot take the Final Exam at that time, you will *fail* Physics 7C, unless arrangements for an incomplete (subject to the instructor's approval) are made *ahead of time*. The following excerpt from the *Winter Quarter 2003 Class Schedule and Room Directory* states the official policy regarding the adjustment of Final Exam times for individual cases (emphasis added):

The Final Exam will be weighted as either 40% or 80% of your course letter grade, whichever is to your best advantage

"The University of California at Davis seeks to accommodate any student who, in observance of a religious creed, encounters on this campus an unavoidable conflict with a test or examination schedule. It is the responsibility of the student to provide, in writing and at the beginning of the quarter, notification of a potential conflict to the individual responsible for administering the test or examination and to request accommodation. Instructors will consider such requests on a case-by-case basis and determine whether such conflicts can be resolved without imposing on the instructor or the other students in the class an undue hardship which cannot be reasonably

Policy regarding the adjustment of Final Exam times, due to religious beliefs

avoided. If so, the instructor shall determine, in consultation with the student, a time during which the student can take the test or examination without incurring a penalty or violating his/her religious creed."

Policy regarding the adjustment of Final Exam times, due to four or more other finals on the same day (not applicable during the Summer Sessions)

The only other circumstances by which a Final Exam may be rescheduled is when there are *four* or more finals scheduled on the *same* day during finals week. (For obvious reasons this policy is in affect only during the regular academic school year, and does *not* apply during the Summer Sessions.)

Policy regarding the binding agreement of an incomplete grade, which may only be arranged in the most extreme circumstances, and which must be completed within one academic year, at a penalty of the assignment of a failing grade and the complete loss of credit

Incompletes are given only in the most extreme of circumstances where it is not possible for you to take the Final Exam at the assigned time and day, and must be approved by the instructor, the Chair of the Physics Department, and/or the Dean of the Division of Mathematical and Physical Sciences in the College of Letters and Sciences. Should you receive an incomplete, you sign a binding contract with the instructor to take the Final Exam with a later cycle of Physics 7C students, *but you do not re-register for this class on RSVP* (you are merely completing the credits you signed up for in this session/quarter). If you do not take the Final Exam within one year of signing the incomplete contract, the Office of the Registrar will arbitrarily assign you a *failing grade* for Physics 7C, in spite of the quantity and quality of your coursework up until the Final Exam. *Remember that incompletes are binding contracts to be arranged solely between yourself and the instructor, and as such can only be completed by arrangement between both parties. Under no circumstances will you be allowed to make up an incomplete signed by one instructor with another instructor.*⁴

Policy regarding the inspection of Final Exams in the following quarter

A Final Exam can be inspected during the following quarter, *but cannot be retroactively submitted for a regrade*. The following excerpt from the *UC-Davis Office of the University Registrar Grade Change Guidelines* states the official policy regarding the retroactive change of grades (emphasis added):

"Academic Senate regulations (system-wide and individual campuses) provide that grades are final when filed with the Registrar by an instructor... Academic Senate regulations prohibit a change of grade based on reevaluation of a student's work or upon the submission of additional work... As evaluations of one person's performance by another person, grades are of necessity,

⁴ No incompletes can be arranged with Dr. Len for Physics 7C, Winter quarter, 2003, as he will no longer be allowed to perform academic duties at any and all University of California campuses after March 21, 2003. Arrangements for incompletes must instead be coordinated directly with the Department of Physics Vice-Chair, Dr. Wendell H. Potter, who will have the sole discretion to unilaterally determine the terms and conditions of your incomplete contract to be fulfilled at a later date, which can and will be substantively different than the policies in effect for Physics 7C in Winter quarter, 2003.

somewhat imprecise, and must be recognized as such. The GCC [Grade Change Committee] recognizes that some grade changes are necessary and is guided by the principle of fairness to the individual student, to the student body in general, and to the faculty. Unfortunately, a number of grade changes are needlessly and futilely sought. Faculty are reminded of their responsibility to be knowledgeable of the regulations regarding grades and to inform students of what is expected of them. Students are reminded of their responsibility to be aware of the procedures and regulations in the General Catalog and Class Schedule and Registration Guide...and to familiarize themselves with the expectations of their instructors."

Discussion/Laboratory Grading

The DL is the central part of this course. *If you fail DL, you will fail Physics 7C.* Your TA will determine your DL grade based on your exit handout homework and participation, and it is your responsibility to comply with the DL policies to be set forth by your TA. How this grade is determined is at the discretion of your TA. Possible DL grades assigned by your TA are:

You must receive a passing DL grade in order to pass Physics 7C

HP	High pass—increases your lecture grade by 0.250 points (on the 4.500 scale). A high pass cannot raise an "A" to an "A+."
P	Pass—does not change your lecture grade. It is expected that 90% of the class will receive this DL grade.
LP	Low pass—decreases your lecture grade by 0.250 points.
U	Unsatisfactory—decreases your lecture grade by 1.000 points (<i>i.e.</i> , a whole letter grade: "B-" to "C-", or "B" to "C", <i>etc.</i>).
F	<i>Fail the course</i> —your lecture grade automatically is dropped to 0.000 points.

DL grades and their effect on your lecture grade in determining your course letter grade

Attendance in DL is mandatory. If for any reason should you miss a DL, please carefully read the following:

Policy regarding making-up a missed DL

- See if it is possible to make up a DL in a later section. The TA in another DL section has the discretion whether to allow you to make-up a DL in his/her section, with space and/or disruption being prime factors to consider. If allowed to make-up a DL, have this TA sign off on your exit handout as written proof to show your regular TA that you made up a DL absence.
- It is your responsibility to document and inform your TA of any DL that you have made-up. Oral testimony is *not* an acceptable replacement for written documentation.

- A make-up DL becomes an excused absence. However, do not abuse the privilege of making up DLs in other sections. You are not allowed to make up more than four DLs with a TA other than your own.
- You are allowed only one unexcused DL absence with no detriment to your DL grade. A DL that is not made up becomes an unexcused absence. A make-up DL without written documentation becomes an unexcused absence. What constitutes an excused absence is at the sole discretion of your TA and instructor(s), together.
- With two or more unexcused DL absences your highest DL grade is a low pass, and your lecture grade will lower by 0.250 points.
- With three unexcused DL absences your highest DL grade is unsatisfactory, and your lecture grade will lower by 1.000 points.
- Four or more DL absences (whether excused or unexcused) is totally unacceptable, and you will *fail* Physics 7C. An unexcused absence also includes (but is not limited to) attendance with chronic tardiness, leaving "early," lack of active participation, incomplete homework, and disruptive behavior.

Academic Misconduct

Student Judicial Affairs handles all referrals of suspected academic misconduct

Cheating of any form is always immediately reported to Student Judicial Affairs for appropriate action. Referrals to Student Judicial Affairs will result in a "Y" hold on your course grade, until the referral has been resolved. Failure to respond to a referral to Student Judicial Affairs to discuss a referral results in an indefinite hold on your entire UC-Davis transcript. As such, please refrain from the temptations and opportunities of academic misconduct.

Any suggestions you may have to help reduce the temptations and opportunities that may result in instances of academic misconduct would be much appreciated.

Policy regarding the reporting and referral of cases of alleged academic misconduct

The following excerpt from the *UC-Davis Office of the University Registrar Student Rights and Records* states the official policy regarding the referral of suspected incidences of academic misconduct (emphasis added):

"The Office of Student Judicial Affairs (SJA) administers the campus student disciplinary system, under authority delegated from the Chancellor. Cases involving alleged violations of University policies or campus rules by students must be referred to this office, which maintains centralized, confidential disciplinary records. Where possible, informal procedures are used, emphasizing the personal growth and development of the student. Where formal procedures are used, the system is designed to provide a prompt, fair, and impartial hearing and resolution of the matter. A student may consult an advisor or attorney at any stage in the informal or formal process. When a complaint is received by Student Judicial Affairs, SJA notifies the student in writing of the alleged misconduct, and directs the student to schedule a meeting with SJA... At the

initial meeting, the student is advised of his/her rights and informed of the evidence supporting the charges. The student is provided with an opportunity to respond and to discuss possible resolutions of the case."

Physics 7C Core Concepts

Notes on Physics 7C Core Concepts for Blocks 11-15

The following pages outline the specific concepts to be covered in lecture and in DL during Winter quarter 2003, to allow you to focus on the idiosyncratic nomenclature and terminology used in this course. Note that these concepts do not comprise the entire contents of the *Physics 7C Course Notes* (W. H. Potter, 2002, J. Wiley Custom Services, ISBN 0-471-23044-8) nor any other third-party introductory physics textbook. *There is no single physics textbook that will exactly correspond to the specific concepts outlined in this course as it is taught during Winter quarter 2003; thus no single textbook will be recommended over any other—if you do choose a particular physics textbook as a reference, these core concepts will assist you in narrowing the scope of your studying for this course.*

Some concepts listed here will be emphasized more than other concepts; additional concepts may be added or deleted as time allows during the progression of this course. ~~The concepts that are included merely for the sake of completeness, and thus will neither appear on a quiz nor the Final Exam are highlighted in gray.~~

Take particular attention to the learning goals and quiz grading criteria for each Block—these should indicate to you as to "what's important" in terms of studying for each quiz, and for determining the breadth and depth of your answers to be written on a typical quiz question. *Remember that in this course, you are graded explicitly on the basis of the completeness and clarity of your discussion, and not necessarily (or at all!) for a numerical answer or final conclusion.* The emphasis of Physics 7 is on conceptual understanding, not on rote memorization nor plug-and-chug calculations!

Block 11 Core Concepts

$$y_{SHM}(t) = A \sin\left(\frac{2\pi t}{T} + \psi_{SHM}\right)$$

$$y(x, t) = A \sin\left(2\pi \frac{t}{T} \pm 2\pi \frac{x}{\lambda} + \psi_{wave}\right)$$

$$y(x, t) = A \exp\left[-\left(\frac{t}{\tau} \pm \frac{x}{\sigma}\right)^2\right]$$

$$\left\{ \begin{array}{l} \frac{d}{dt} A \sin(\beta t) = \beta A \cos(\beta t) \\ \frac{d}{dt} A \cos(\beta t) = -\beta A \sin(\beta t) \end{array} \right.$$

$$\left\{ \begin{array}{l} \frac{d}{dt} e^{\beta t} = \beta e^{\beta t} \\ \frac{d}{dt} e^{f(t)} = e^{f(t)} \cdot \frac{d}{dt} f(t) \end{array} \right.$$

$$\left\{ \begin{array}{l} v_y = \frac{d}{dt} y(t) \\ a_y = \frac{d}{dt} v_y(t) = \frac{d}{dt} \left(\frac{d}{dt} y(t) \right) \end{array} \right.$$

$$T_{SHM} \propto \sqrt{\frac{\text{inertial parameter}}{\text{restoring parameter}}}$$

$$T_{\text{pendulum}} = 2\pi \sqrt{\frac{L}{g}}$$

$$T_{\text{mass-spring}} = 2\pi \sqrt{\frac{m}{k}}$$

$$f = \frac{1}{T}$$

$$v_{\text{particle}}(x, t) = \frac{d}{dt} y_{\text{particle}}(x, t)$$

Displacement $y(t)$
of a simple harmonic oscillator

Displacement $y(x, t)$
of a harmonic wave

Displacement $y(x, t)$
of a pulse wave

Trigonometric derivatives

Exponential derivatives

Velocity $\left[\frac{\text{m}}{\text{s}}\right]$ and acceleration $\left[\frac{\text{m}}{\text{s}^2}\right]$
as successive time derivatives of
displacement $y(t)$

Generalized period for all ideal SHM
systems

Period [s] of a pendulum SHM
system

Period [s] of a mass-spring SHM
system

Relation between frequency [Hz] and
period [s]

Displacement velocity $\left[\frac{\text{m}}{\text{s}}\right]$ of a
particle in a medium, due to a wave

$$\Psi(x, t) = \left(2\pi \frac{t}{T} \pm 2\pi \frac{x}{\lambda} + \Psi_{\text{wave}} \right)$$

Total phase angle for harmonic waves
[rad]

$$v_{\text{wave}} \propto \sqrt{\frac{\text{restoring parameter}}{\text{inertial parameter}}}$$

Idealized dependence of wave velocity
 $\left[\frac{\text{m}}{\text{s}} \right]$ on properties of the medium

$$v_{\text{wave, sound}} = \sqrt{\frac{B}{\rho}}$$

Sound wave velocity $\left[\frac{\text{m}}{\text{s}} \right]$

$$v_{\text{wave, rope}} = \sqrt{\frac{F_{\text{tension}}}{\mu}}$$

Rope wave velocity $\left[\frac{\text{m}}{\text{s}} \right]$

$$v_{\text{wave, light}} = \frac{c}{n_{\text{medium}}} = \frac{(3.00 \times 10^8 \text{ m/s})}{n_{\text{medium}}}$$

Light wave velocity $\left[\frac{\text{m}}{\text{s}} \right]$

$$v_{\text{wave, water}} = \sqrt{\frac{g\lambda}{2\pi} \tanh\left(2\pi \frac{d}{\lambda}\right)} \approx \sqrt{\frac{g\lambda}{2\pi}}$$

Water wave velocity $\left[\frac{\text{m}}{\text{s}} \right]$
(with deep water approximation)

$$\lambda = \frac{v_{\text{wave}}}{f} = v_{\text{wave}} T$$

Wavelength [m]

Block 11 Learning Goals

- Identifying relevant parameters to describe simple harmonic motion or harmonic waves.
- Graphically representing simple harmonic motion or harmonic waves.
- Connection between motion of particles in medium, and the motion of the wave through the particles in the medium itself.
- Understanding relation between independent and dependent wave parameters.
- Modeling different wave phenomena as ideal harmonic waves.

Quiz 11 Grading Criteria

- Proper and relevant equation and parameters.
- Proper and relevant graphical representation.
- Isolates independent and dependent wave parameters.
- Neatness, clarity and completeness of discussion and mathematical steps (not every intermediate step needs to be written out).
- Conclusions or numerical answers clearly circled.

Block 12 Core Concepts

$$y_{total} = y_1 + y_2$$

$$y(L, t) = A \sin \left[2\pi \frac{t}{T} - 2\pi \frac{L}{\lambda} + \underbrace{\left(\Psi_{source} + \Psi_{reflection} \right)}_{\Psi_{wave}} \right]$$

$$\begin{aligned} \Delta\Psi &= \Psi_1 - \Psi_2 \\ &= \left(2\pi t(\Delta f) - 2\pi \Delta \left(\frac{L}{\lambda} \right) + \Delta\Psi_{sources} + \Delta\Psi_{reflections} \right) \end{aligned}$$

$$\Delta L = L_1 - L_2$$

$$\Delta L = \begin{cases} (\#)\lambda & \text{constructive} \\ \left(\# + \frac{1}{2}\right)\lambda & \text{destructive} \end{cases}$$

$$\Delta L = \begin{cases} \left(\# + \frac{1}{2}\right)\lambda & \text{constructive} \\ (\#)\lambda & \text{destructive} \end{cases}$$

$$\Delta\Psi = \begin{cases} \pm(\text{even})\pi & \text{constructive} \\ \pm(\text{odd})\pi & \text{destructive} \end{cases}$$

$$\Psi_{reflection} = \begin{cases} 0 & \text{"soft" reflection} \\ \pi & \text{"hard" reflection} \end{cases}$$

$$n_{medium} = \frac{c}{v_{medium}}$$

$$\Delta\Psi = \left(2\pi t \underbrace{(f_1 - f_2)}_{f_{beat}} + [constant] \right) = \begin{cases} \pm(\text{even})\pi & \text{constructive} \\ \pm(\text{odd})\pi & \text{destructive} \end{cases}$$

$$f_{beat} = f_1 - f_2$$

Wave superposition

Displacement $y(x, t)$ of a harmonic wave, in terms of path-length L

Total phase difference [rad]

Path-length difference [m]

Interference condition for two *in-phase* sources of the *same* period/wavelength

Interference condition for two *out-of-phase* sources of the *same* period/wavelength

Generalized interference condition for *any* two waves of *any* period/wavelength

Reflection phase shifts

"Index of refraction" for light waves

Time-dependent interference condition for two waves of the different periods/wavelengths ("beats")

Beat frequency [Hz]

$$f_{\text{carrier}} = \frac{1}{2}(f_1 + f_2)$$

Carrier frequency [Hz]

$$\Delta L = d \sin \theta = \begin{cases} (\#)\lambda & \text{constructive} \\ (\# + \frac{1}{2})\lambda & \text{destructive} \end{cases}$$

Interference condition for double-slit interference

$$f_1 = \frac{v_{\text{wave}}}{2L}; f_{\#} = (\#)f_1$$

Fundamental and harmonic frequencies for a *symmetric* resonant standing wave system

$$f_1 = \frac{v_{\text{wave}}}{4L}; f_{\text{odd}} = (\text{odd})f_1$$

Fundamental and harmonic frequencies for an *asymmetric* resonant standing wave system

$$\Delta L = 2t = \begin{cases} (\# + \frac{1}{2})\lambda_2 & \text{constructive} \\ (\#)\lambda_2 & \text{destructive} \end{cases}$$

Interference condition for a high impedance medium separating two lower impedance media (or *vice versa*)

$$\Delta L = 2t = \begin{cases} (\#)\lambda_2 & \text{constructive} \\ (\# + \frac{1}{2})\lambda_2 & \text{destructive} \end{cases}$$

Interference condition for a stack of consecutively increasing impedance media (or *vice versa*)

$$\Delta \Psi = \left(-2\pi\Delta\left(\frac{L}{\lambda}\right) + \Delta\Psi_{\text{reflections}} \right) = \begin{cases} \pm(\text{even})\pi & \text{constructive} \\ \pm(\text{odd})\pi & \text{destructive} \end{cases}$$

Generalized interference condition for *any* system of relative impedance media

Block 12 Learning Goals

- Analyzing interference phenomena, and be able to use the most generalized interference conditions applicable.
- Understanding how interference phenomena is generated and measured experimentally.
- Understanding how the most generalized interference conditions revert to simpler forms under very specific conditions.

Quiz 12 Grading Criteria

- Proper and relevant equation and parameters.
- Proper and relevant graphical representation.
- Isolates independent and dependent wave parameters.
- Neatness, clarity and completeness of discussion and mathematical steps (not every intermediate step needs to be written out).
- Conclusions or numerical answers clearly circled.

Block 13 Core Concepts

Object A $\overset{\text{exerts}}{\underset{\text{force}}{\Rightarrow}}$ object B

Direct model of forces

$$\vec{F}_{\text{gravity of } M \text{ on } m} = \begin{cases} \text{magnitude} = G \frac{Mm}{(r_{M \leftrightarrow m})^2} \\ \text{direction} = \text{in towards } M \end{cases}$$

Direct model of gravitational forces

$$\vec{F}_{\text{electric of } Q \text{ on } q} = \begin{cases} \text{magnitude} = k \frac{Qq}{(r_{Q \leftrightarrow q})^2} \\ \text{direction} = \begin{cases} \text{attractive if } \pm q, \mp Q \\ \text{repulsive if } \pm q, \pm Q \end{cases} \end{cases}$$

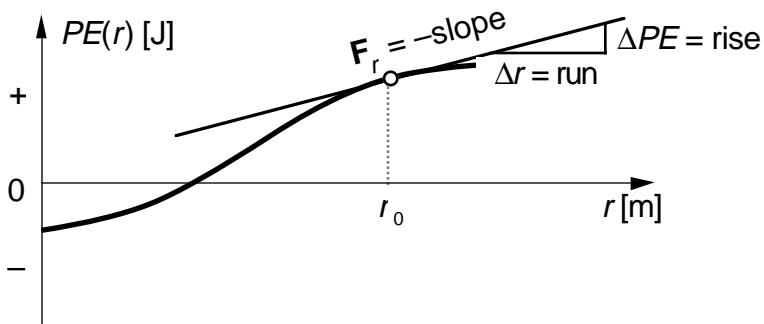
Direct model of electric forces

$$\Delta PE_{\text{grav}} = -GMm \Delta \left(\frac{1}{r} \right) = -GMm \left(\frac{1}{r_{\text{final}}} - \frac{1}{r_{\text{initial}}} \right)$$

Changes in gravitational potential energy [J]

$$\Delta PE_{\text{elec}} = kQq \Delta \left(\frac{1}{r} \right) = kQq \left(\frac{1}{r_{\text{final}}} - \frac{1}{r_{\text{initial}}} \right)$$

Changes in electric potential energy [J]



$$F_{\text{along } r} = - \frac{\Delta PE}{\Delta r}$$

Gradient relation between force and potential energy

Object A $\overset{\text{creates}}{\Rightarrow}$ field $\overset{\text{exerts}}{\underset{\text{force}}{\Rightarrow}}$ object B

Field model of forces

$$\left. \begin{array}{l} \vec{\mathbf{g}}_M \left\{ \begin{array}{l} \text{magnitude} = \left| G \frac{M}{r^2} \right| \\ \text{direction} = \text{in towards } M \end{array} \right\} \\ \vec{\mathbf{F}}_{\text{on } M} \left\{ \begin{array}{l} \text{magnitude} = |mg| \\ \text{direction} = \text{along } \vec{\mathbf{g}} \end{array} \right\} \end{array} \right\} \text{Field model of gravitational forces}$$

$$\left. \begin{array}{l} \vec{\mathbf{E}}_Q \left\{ \begin{array}{l} \text{magnitude} = \left| k \frac{Q}{r^2} \right| \\ \text{direction} = \begin{pmatrix} \text{in towards } -Q \\ \text{out away from } +Q \end{pmatrix} \end{array} \right\} \\ \vec{\mathbf{F}}_{\text{on } q} \left\{ \begin{array}{l} \text{magnitude} = |qE| \\ \text{direction} = \begin{pmatrix} \text{along } \vec{\mathbf{E}} \text{ for } +q \\ \text{opposite } \vec{\mathbf{E}} \text{ for } -q \end{pmatrix} \end{array} \right\} \end{array} \right\} \text{Field model of electric forces}$$

$$\left. \begin{array}{l} \vec{\mathbf{B}}_I \left\{ \begin{array}{l} \text{magnitude} = \left| \frac{\mu_0 I}{2\pi r} \right| \\ \text{direction} = \text{RHR1} \end{array} \right\} \\ \vec{\mathbf{F}}_{\text{on } qv} \left\{ \begin{array}{l} \text{magnitude} = |qvB \sin \theta_v^B| \\ \text{direction} = \begin{pmatrix} \text{along RHR2 for } +q \\ \text{opposite RHR2 for } -q \end{pmatrix} \end{array} \right\} \end{array} \right\} \text{Field model of magnetic forces}$$

Block 13 Learning Goals

- Identification of source and test objects.
- Modeling gravitational/electric/magnetic interactions with the direct model of forces.
- Modeling gravitational/electric/magnetic interactions with the field model of forces.
- Modeling inter-mass/inter-charge/inter-atomic interactions with potential energy graphs.
- Understanding the relations between forces and fields, and forces and potential energy gradients.
- Superposition of force vectors, and superposition of field vectors.

Quiz 13 Grading Criteria

- Proper and relevant equation and parameters.
- Proper and relevant graphical representation.
- Vector addition (tail-to-head or component-by-component) clearly shown.
- Appropriate choice of method (direct or field) used in best describing the interaction between source and test object.
- Neatness, clarity and completeness of discussion and mathematical steps (not every intermediate step needs to be written out).
- Conclusions or numerical answers clearly circled.

Block 14 Core Concepts

$${}^A_Z X \begin{cases} X = \text{periodic table name of element} \\ A = \text{number of neutrons and protons} \\ Z = \text{number of protons} \end{cases}$$

Notation for a nucleus

$$R = (1.2 \text{ fm})A^{1/3}$$

Radius of a nucleus

$$\begin{aligned} \text{"Binding energy"} &= (\text{mass defect})c^2 \\ &= (m_{\text{nucleus}} - m_{\text{protons}} - m_{\text{neutrons}})c^2 \end{aligned}$$

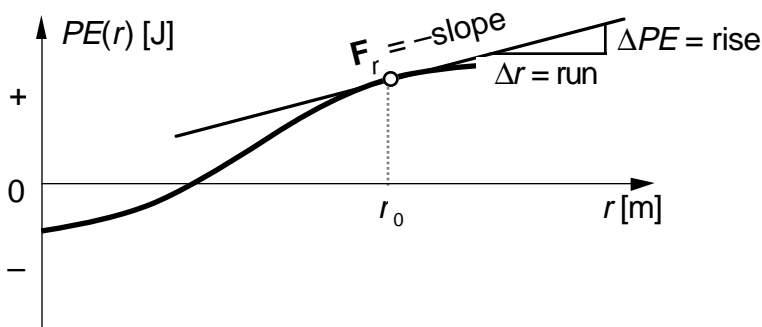
Binding energies and mass defects

$$\begin{aligned} \text{"Q - value"} &= (\text{mass decrement})c^2 \\ &= (m_f - m_i)c^2 \end{aligned}$$

Q-values and mass decrements

$$XE_{\text{photon}} = hf_{\text{photon}} = \frac{hc}{\lambda_{\text{photon}}}$$

Photon frequency/wavelength



$$F_{\text{along } r} = -\frac{\Delta PE}{\Delta r}$$

Gradient relation between force and potential energy

$$\vec{F}_{\text{electric of } Q \text{ on } q} = \begin{cases} \text{magnitude} = k \frac{Qq}{(r_{Q \leftrightarrow q})^2} \\ \text{direction} = \begin{pmatrix} \text{attractive if } \pm q, \mp Q \\ \text{repulsive if } \pm q, \pm Q \end{pmatrix} \end{cases}$$

Direct model of electric forces

$$\Delta PE_{\text{elec}} = kQq\Delta\left(\frac{1}{r}\right) = kQq\left(\frac{1}{r_{\text{final}}} - \frac{1}{r_{\text{initial}}}\right)$$

Changes in electric potential energy [J]

Block 14 Learning Goals

- Identification of source and test objects.
- Modeling inter-nuclear interactions with potential energy gradients.
- Modeling weak force interactions with the box model of nuclear energy levels.
- Applying mass-energy conservation to nuclear processes.
- Analyzing nuclear fusion/fission processes, and decay (α , β , and γ) processes.

Quiz 14 Grading Criteria

- Proper and relevant equation and parameters.
- Proper and relevant graphical representation.
- Identification and modeling of relevant nuclear processes.
- Neatness, clarity and completeness of discussion and mathematical steps (not every intermediate step needs to be written out).
- Conclusions or numerical answers clearly circled.

Block 15 Core Concepts

$$\theta_1 = \theta_2$$

Law of Reflection
("Hero's Law")

$$n_{\text{medium}} = \frac{c}{v_{\text{medium}}}$$

Index of refraction

$$n_1 \sin \theta_1 = n_2 \sin \theta_2$$

Snell's Law
("Descartes' Law")

$$\theta_c = \text{Arc sin} \left(\frac{n_2}{n_1} \right)$$

Critical angle

$$\theta_1 > \theta_c$$

Condition for TIR
(total internal reflection)

$$\frac{1}{o} + \frac{1}{i} = \frac{1}{f}$$

Thin lens equation
("Thin mirror equation")

$$M_{\text{linear}} = \frac{h_i}{h_o} = - \left(\frac{i}{o} \right)$$

Linear magnification

$$D = \frac{1}{f}$$

Optical strength [diopters]

Block 15 Learning Goals

- Understanding how light reflects, refracts, or totally internally reflects at the interface between media of different relative indices of refraction.
- Modeling optical systems as thin lenses, in the formation of images.
- Understanding object/image placement for converging/diverging lenses, modeled by the thin lens equation or by ray tracing.
- Treating multiple lens systems as sequential object→lens→image processes.
- Understanding how eyesight defects can be corrected by use of glasses or contact lenses.

Quiz 15 Grading Criteria

- Proper and relevant equation and parameters.
- Proper and relevant graphical representation.
- Identification and modeling of relevant optical systems.
- Methodical approach to multiple-lens systems.
- Neatness, clarity and completeness of discussion and mathematical steps (not every intermediate step needs to be written out).
- Conclusions or numerical answers clearly circled.

Block 11 Glossary

Oscillations glossary

acceleration a_y

amplitude A

constant phase angle ψ_{SHM}

derivatives (trigonometric)

displacement y, θ

equilibrium

frequency f

gravitational field

constant g

Hooke's Law

mass-spring SHM

oscillations

pendulum SHM

period T

simple harmonic motion (SHM)

spring constant k

velocity v_y

acceleration a_y

A *vector* that describes the change in the **velocity** of an object with respect to time—its *magnitude* (measured in units of m/s^2) is determined from the first time **derivative** of the velocity of the object, or the second derivative of the **displacement** of the object (its *direction* is simply the direction of the net force vector on the object).

$$a_y = \begin{cases} \text{magnitude} = \frac{d}{dt} v_y(t) = \frac{d}{dt} \left(\frac{d}{dt} y(t) \right) \left[\frac{\text{m}}{\text{s}^2} \right] \\ \text{direction} = \text{direction of } \sum \vec{\mathbf{F}} \end{cases}$$

(See **derivatives (trigonometric)**, **displacement**, **velocity**.)

amplitude A

The sine function of a **simple harmonic motion** object varies between -1 and $+1$. The value of the amplitude A determines the **displacement** values that the SHM object oscillates about, between $-A$ and $+A$, as measured from **equilibrium**. (See **displacement**, **equilibrium**, **simple harmonic motion**.)

constant phase angle ψ_{SHM}

This is basically where (on the unit circle) an SHM object starts in its periodic cycle at $t = 0$. This parameter must be in units of radians. (*N.b.*: This is lower-case "psi" (we point this out now in order to distinguish this from upper-case "psi" Ψ , which is used for the *total phase angle* of harmonic functions.) (See **simple harmonic motion**.)

derivatives (trigonometric)

The derivative of a sine function, with respect to time, gives a cosine function—but don't forget to apply the chain rule to take the derivative of what was inside the sine function as well:

$$\frac{d}{dt} A \sin(\beta t) = \beta A \cos(\beta t).$$

The derivative of a cosine function, with respect to time, gives a negative sine function—but don't forget to apply the chain rule to take the derivative of what was inside the cosine function as well:

$$\frac{d}{dt} A \cos(\beta t) = -\beta A \sin(\beta t).$$

(See **velocity, acceleration**.)

displacement y, θ

Where the position of an oscillating object is, with respect to its **equilibrium**. If we are talking about a mass on a spring oscillating back and forth, we would be measuring y in meters. If we are talking about a mass on a string swinging to and fro, we would be measuring θ in degrees (or radians). (See **equilibrium, simple harmonic motion**.)

equilibrium

The location of an SHM object, when it is stationary and not oscillating. This defines where the **amplitude** of its motion is measured from. (See **amplitude, simple harmonic motion**.)

frequency f

(See **period**.)

gravitational field constant (g)

The force of gravity of a planet on an object (at or near its surface) is given by:

$$\vec{\mathbf{F}}_{\text{gravity of planet on } m} = \begin{cases} \text{magnitude} = |mg_{\text{planet}}| \\ \text{direction} = \text{downwards} \end{cases}$$

where g is the gravitational field constant of that planet (at or near its surface). This constant has a value of approximately $g_{\text{Earth}} \approx 9.8 \text{ N/kg}$ at/near the surface of the Earth, and $g \rightarrow 0$ at an infinite distance away from the center of the Earth. Other planets will have different g values at/near their surfaces.

Hooke's Law

An object is said to "obey" **Hooke's Law** if the following conditions regarding its *net force* $\sum \mathbf{F}$ are met:

- The magnitude of the net force on an object is proportional to the **displacement** y of the object from a well-defined **equilibrium** position.
- The direction of the net force on an object points in the direction back towards this equilibrium position (hence the negative sign).
- The constant of proportionality between the net force and displacement y is given by k .

In vector equation form, Hooke's Law is given by:

$$\sum \vec{\mathbf{F}} = -k\vec{y}.$$

A Hooke's Law net force on an object causes it to undergo **simple harmonic motion**. An object undergoing simple harmonic motion has a net force that is described by Hooke's Law. This reasoning may sound circular, but keep in mind that this is due to the degree that Hooke's Law net forces and time-dependent sinusoidal behaviors are intertwined. You can't have one without the other!

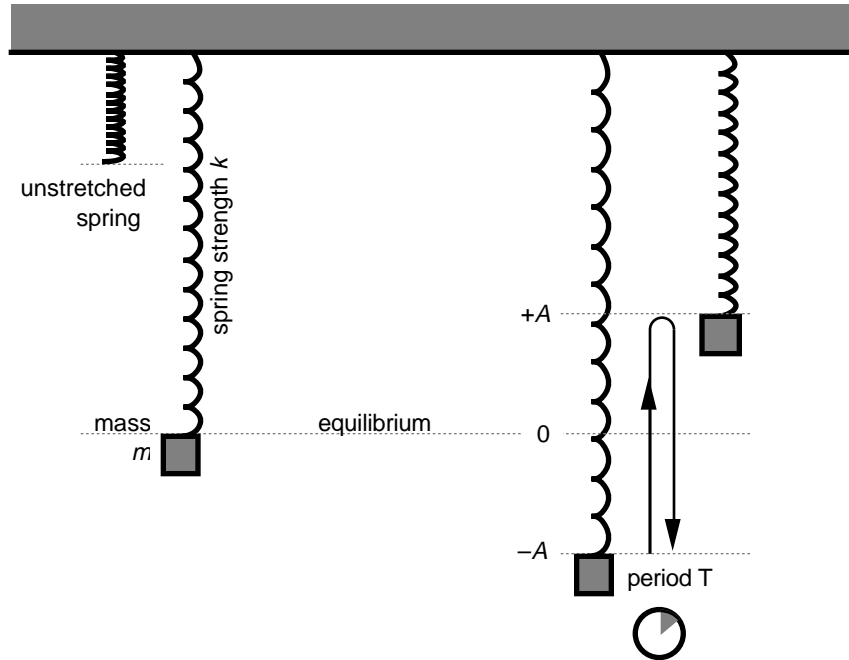
Note that similar arguments can be made regarding the rotational simple harmonic motion of an object, where the constant of proportionality between the *net torque* and *rotational displacement* is given by κ (lower-case Greek letter "kappa"):

$$\sum \vec{\tau} = -\kappa\vec{\theta}.$$

(See **displacement, equilibrium, simple harmonic motion**.)

mass-spring SHM

An object of mass m attached to a spring with a **spring constant** k that is allowed to freely oscillate with no friction will undergo **simple harmonic motion**.



There can be horizontal or vertical mass-spring SHM systems. In either case, the *net force* $\sum \mathbf{F}$ on the object must obey the translational form of **Hooke's Law** in order for the object to undergo SHM.

The **amplitude** A of its motion is the measured from **equilibrium**.

The **period** T of a mass-spring system depends only upon the mass m and spring constant k :

$$T_{\text{mass-spring}} = 2\pi\sqrt{\frac{m}{k}}.$$

(See **amplitude**, **Hooke's Law**, **equilibrium**, **period**, **simple harmonic motion**, **spring constant**.)

oscillations

Any system that experiences a cyclical **displacement** from **equilibrium**, *e.g.*, a mass that moves back and forth. Note that all **simple harmonic motion** systems are oscillatory, but not all oscillating systems are simple harmonic motion. (See **displacement**, **equilibrium**, **simple harmonic motion**.)

pendulum SHM

An object of mass m attached to a string with strength L , in a location with a **gravitational field constant** g that is allowed to freely swing back-and-forth with no friction will undergo **simple harmonic motion**.

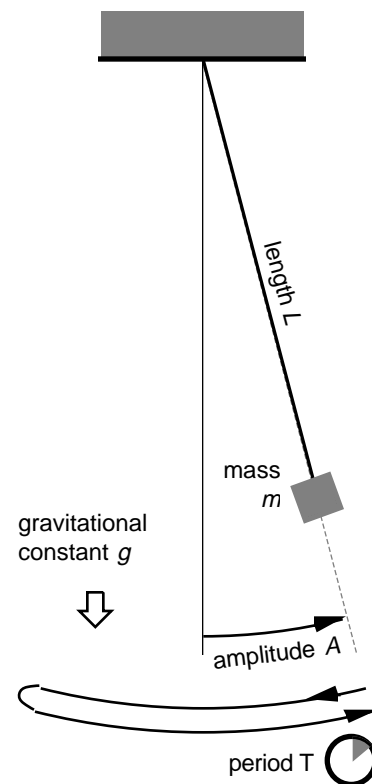
Note that this is a rotational system, rather than a translational xyz system. Instead of considering the *net force* $\sum \mathbf{F}$ on the object, we actually consider the *net torque* $\sum \tau$ on the object. In any case, the net torque on the object must obey the rotational form of **Hooke's Law** in order for the object to undergo SHM.

The **amplitude** A of its motion is the angle measured from **equilibrium** (usually vertically straight down).

The **period** T of a pendulum system depends only upon the length L of the string, and gravitational field constant g :

$$T_{\text{pendulum}} = 2\pi \sqrt{\frac{L}{g}}.$$

(See **amplitude**, **Hooke's Law**, **equilibrium**, **gravitational field constant**, **period**, **simple harmonic motion**.)



period T

This is how much time (in seconds) it takes for a **simple harmonic motion** object to complete one cycle of periodic motion. Related to the period T is the **frequency** f , which is how many cycles of periodic motion that the SHM object undergoes in one second (in units of cycles/second, or "Hertz"):

$$f = \frac{1}{T}.$$

Most generally, for *all* ideal simple harmonic motion systems, the period T is proportional to a square root of a *inertial* parameter, and inversely proportional to the square root of an *restoring* parameter:

$$T_{SHM} \propto \sqrt{\frac{\text{inertial parameter}}{\text{restoring parameter}}}.$$

(See **frequency**, **simple harmonic motion**.)

simple harmonic motion (SHM)

An object whose oscillating **displacement** is described by a time-dependent sine function ("harmonic" is synonymous with sine/cosine trigonometric functions), and whose *net force* obeys

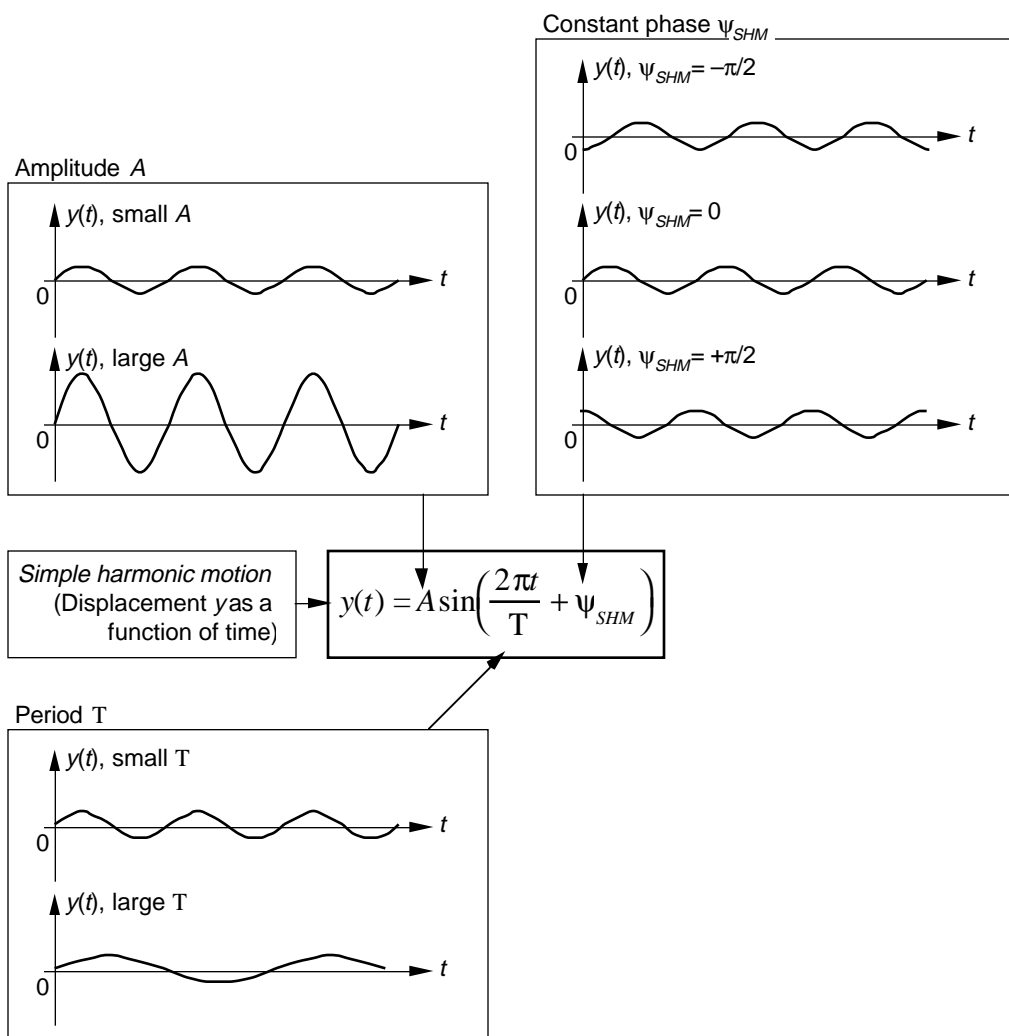
Hooke's Law. The general equation for the displacement of any SHM object as a function of time t is given by:

$$y_{SHM}(t) = A \sin\left(\frac{2\pi t}{T} + \psi_{SHM}\right).$$

The following parameters make up the SHM function for this object, and are defined elsewhere in this glossary:

- **displacement** y .
- **amplitude** A .
- **period** T .
- **constant phase** ψ_{SHM} .

Note that the $y(t)$ function takes the "sine of" something in radian units! This is why the 2π factor is included inside the $y(t)$ function—when we stick in a time t , the $(2\pi t/T)$ term gets converted



into radian units (the seconds units cancel), such that we wind up taking the sine of radians (as opposed to taking the sine of degrees). Make sure you know how to set your calculator in "RAD" mode instead of "DEG" mode...

The dependence of the SHM displacement function on these parameters is schematically depicted on the previous page. (See **amplitude, constant phase, displacement, period.**)

spring constant k

The "strength" of a spring. If a spring requires a certain amount of force to stretch it a given distance, then its "strength" k is given by:

$$k = \left| \frac{F_{stretch}}{stretch\ distance} \right|.$$

The units of k are Newtons/meter.

velocity v_y

A *vector* that describes the change in the **displacement** of an object with respect to time—its *magnitude* (measured in units of m/s) is determined from the first time **derivative** of the displacement (its *direction* is simply the direction that the object is heading).

$$v_y = \begin{cases} \text{magnitude} = \frac{d}{dt} y(t) \left[\frac{\text{m}}{\text{s}} \right] \\ \text{direction} = \text{direction of motion} \end{cases}$$

(See **displacement, derivatives (trigonometric).**)

Waves glossary

amplitude A	period T
constant phase angle ψ_{wave}	polarization
"coordinated motion"	pulse wave
dependent wave parameter	"snapshot"
derivatives (exponential)	total phase angle Ψ
displacement y	wave velocity v_{wave}
equilibrium	waves (general)
frequency f	waves, pressure/density ("sound")
harmonic wave	waves, rope
independent wave	waves, TEM ("light")
parameters	waves, water
location x	wavelength λ
medium	
particle velocity $v_{particle}(x,t)$	

amplitude A

The sine function of a **harmonic wave** varies between -1 and $+1$. The value of the amplitude A determines the displacement values that it oscillates about, between $-A$ and $+A$, as measured from **equilibrium**. For a **pulse wave**, the amplitude determines the displacement values that it varies about, between 0 and $+A$ (or 0 and $-A$), as measured from equilibrium. (See **harmonic wave**, **pulse wave**, **equilibrium**.)

constant phase angle ψ_{wave}

This is "where" a **harmonic wave** "is" with respect to its periodic cycle at $t = 0$. If the harmonic wave (with respect to x) is represented by a sine function at $t = 0$, then $\psi_{wave} = 0$. If the wave is a "shifted over" sine function at $t = 0$, then we have to be careful to specify what non-zero value ψ_{wave} is. Remember that this parameter (whatever value it may have) must be in units of radians. (*N.b.*: Don't confuse this lower-case "psi" ψ_{wave} for a harmonic wave with the lower-case "psi" ψ_{SHM} for SHM objects.) (See **harmonic wave**.)

"coordinated motion"

As a **harmonic wave** or a **pulse wave** travels through a **medium** (given by a $y(x,t)$ wave function), a particle at one specific **location** x in the medium undergoes a unique time-varying motion described by the *displacement* function $y(t)$ that is "collapsed" from the original $y(x,t)$ wave function.

For a harmonic wave moving to the *left*, the $y(x,t)$ wave function is given by:

$$y(x,t) = A \sin\left(2\pi \frac{t}{T} + 2\pi \frac{x}{\lambda} + \psi_{\text{wave}}\right).$$

The $y(t)$ motion of a particle at one specific location x_0 in the medium is found by substituting in the value $x = x_0$ into the $y(x,t)$ equation:

$$\begin{aligned} y(x_0,t) &= y(x,t)|_{x=x_0}, \\ &= A \sin\left(2\pi \frac{t}{T} + \underbrace{2\pi \frac{x_0}{\lambda} + \psi_{\text{wave}}}_{\psi_{SHM}}\right) \\ &= A \sin\left(2\pi \frac{t}{T} + \psi_{SHM \text{ at } x=x_0}\right) \end{aligned}$$

If this is done for the particles at *each* and *every* location x in the medium, then they will all have different constant phases, and thus different $y(t)$ motions. However, the $y(t)$ motions of each of these particles will be "coordinated" with each other, as the wave passes through their locations.

This "collapse" of a $y(x,t)$ wave function to the $y(t)$ motion of a particle in a medium can be similarly done for pulse waves as well. (See **harmonic wave, location, medium, pulse wave, "snapshot."**)

dependent wave parameter

A **wave** parameter that depends on *both* the properties of a source *and* the properties of the **medium**. The *only* dependent wave parameter is **wavelength** λ . (See **medium, independent wave parameters, waves, wavelength.**)

derivatives (exponential)

The derivative of an exponential function, with respect to time, gives the same exponential function (multiplied by the constant β from the exponent):

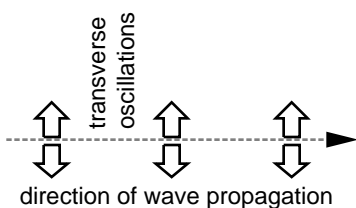
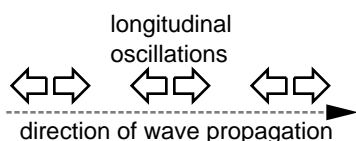
$$\frac{d}{dt}e^{\beta t} = \beta e^{\beta t}.$$

However, if the exponent is a non-trivial function of time, don't forget to apply the chain rule to take the derivative of what was raised in the exponent as well:

$$\frac{d}{dt} e^{f(t)} = e^{f(t)} \cdot \frac{d}{dt} f(t).$$

displacement y

A measure of how much a **medium** has been displaced from its **equilibrium**, as a **wave** travels through it. We could measure y either of two ways, depending on the *polarization* of the wave:



- *Longitudinal polarization*: oscillations of the particles in the medium are *along* the direction of wave velocity. So y is actually in the *same direction* as x , and particles in the medium are displaced back and forth along the y axis (!) as a wave travels along the x axis. Think of a large group of people crowded very close to each other. If people in the back start shoving and pushing, then these back-and-forth shoves are longitudinal waves that propagate through the crowd of people.
- *Transverse polarization*: oscillations of the particles in the medium are in the direction perpendicular (*sideways*) to wave velocity. So y is in the direction *sideways* to x , and particles in the medium are displaced side-to-side as a wave travels along the x axis. Think of making transverse waves along a horizontal length of jump rope by wiggling it up-and-down or side-to-side.

(See **equilibrium, medium, waves**.)

equilibrium

The location or value of a point in a **medium**, when it is stationary and not oscillating (due to the lack of a **wave**, or due to being momentarily stationary). This defines what the **amplitude** of the wave is measured from. (See **amplitude, medium, waves**.)

frequency f

(See **period**.)

harmonic wave

A **wave** whose y displacements at various x locations at t times is given by a general *harmonic wave function*:

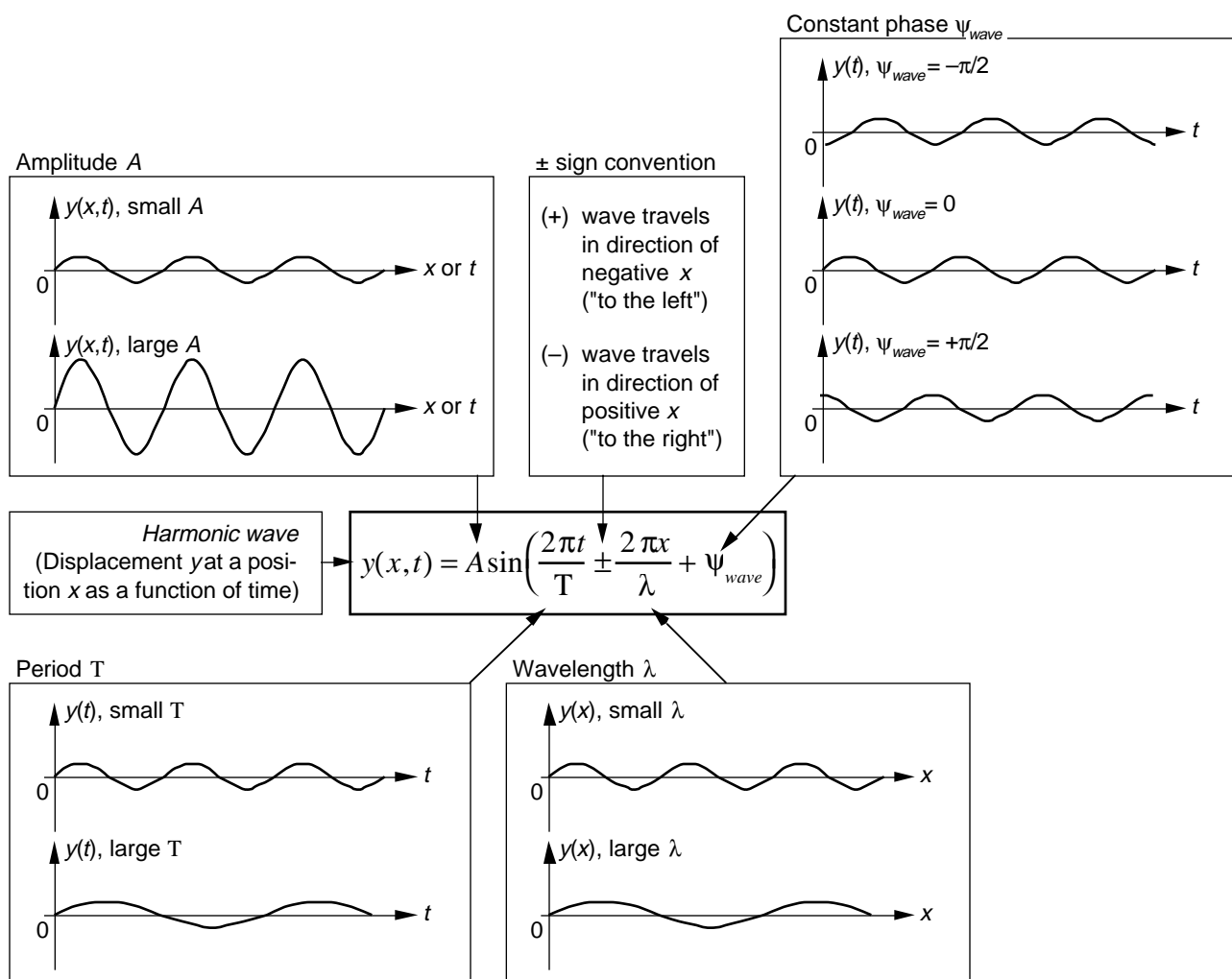
$$y(x, t) = A \sin \left(2\pi \frac{t}{T} \pm 2\pi \frac{x}{\lambda} + \psi_{\text{wave}} \right).$$

Keep in mind that a harmonic wave must be created by an SHM source—this is why both harmonic waves and SHM are both described by harmonic equations (*i.e.*, sine functions) that have similar parameters.

The following parameters make up the harmonic wave function, and are defined elsewhere in this glossary:

- **displacement** y .
- **amplitude** A .
- **period** T .
- **location** x .
- **wavelength** λ .
- **constant phase** ψ_{wave} .

Note that the $y(x,t)$ function takes the "sine of" something in radian units! This is why 2π factors are included inside the $y(x,t)$ function—when we stick in a time t , the $(2\pi t/T)$ term gets converted into radian units (the seconds units cancel), such that we wind up taking the sine of radians (as opposed to taking the sine of degrees). Also, when we stick in a location x , the $(2\pi x/\lambda)$ term gets converted into radians as well. Make sure you know how to set your calculator in "RAD" mode instead of "DEG" mode...



Regarding the (\pm) sign inside the sine function—if we choose the ($-$) sign, the motion of the wave is in the positive x -direction (and thus travels to the right); if the ($+$) sign is chosen, the motion of the wave is in the negative x -direction (and thus travels to the left). Deal with it or it will deal with you...

The dependence of the harmonic wave function on these parameters is schematically depicted on the previous page. (See **amplitude, constant phase, displacement, location, period, waves (general)**.)

independent wave parameters

Those **wave** parameters that depend on either just the properties of a source, or just the properties of the **medium** (*i.e.*, they depend on only one thing, and are independent of everything else *but* that one thing).

The independent wave parameters that depend only on the properties of the *source* are **amplitude** A , **displacement polarization** direction (whether *longitudinal* or *transverse*), **period** T , and **constant phase angle** ψ_{wave} .

The one independent parameter that depends only on the properties of the medium is the **wave velocity** v_{wave} . (See **amplitude, constant phase, dependent wave parameter, displacements, period, wave velocity, waves (general)**.)

"light"

(See **waves (TEM)**.)

location x

The specific position in a **medium** that a wave travels through. A one-dimensional **harmonic wave** or **pulse wave** must either travel along the $+x$ or the $-x$ direction through a medium. (See **harmonic wave, medium, pulse wave**.)

medium

Any system of particles that interact via forces that obey **Hooke's Law**, and thus are subject to coordinated SHM behavior. (See **waves**.)

particle velocity $v_{particle}(x,t)$

Since we have an explicit **harmonic wave** $y(x,t)$ expression for the **displacement** y of a particle at **location** x in a medium, as a function of time, we can take the derivative of this $y(x,t)$ expression with respect to time to find that particle's velocity $v_{particle}(x,t)$:

$$v_{particle}(x,t) = \frac{d}{dt} y_{particle}(x,t)$$

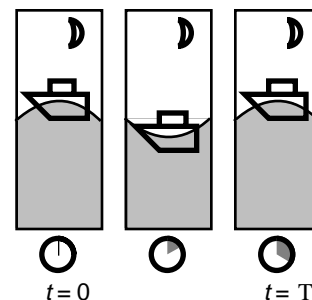
Note that the velocity $v_{particle}(x,t)$ of a particle in a medium, as a wave travels through it is very different from the **wave velocity** v_{wave} through that medium. Think of **water waves** (of a given **wavelength** λ) that move steadily through the ocean in the horizontal direction at a constant velocity v_{wave} . But a buoy located at a specific x position will bob up and down as these waves travel past its x location—the vertical velocity $v_{particle}(x,t)$ of its motion will vary with respect to time, and is distinct from the constant horizontal velocity of the water waves! (See **displacement, harmonic wave, location, wave velocity, wavelength, waves (water).**)

period T

How much time (in seconds) it takes for a particle at a specific location x in a **medium** to complete one cycle of periodic **displacement**, as a **harmonic wave** passes through it. Think of being on an anchored boat that continuously goes up and down because of **water waves**. The period T is the time it takes for your boat to execute one complete up and down oscillation. Don't get seasick! Related to the period T is the **frequency** f , which is how many cycles of periodic motion that a specific position x in a medium undergoes in one second (in units of cycles/second, or "Hertz"):

$$f = \frac{1}{T}.$$

Note that the period for a harmonic wave ultimately depends on the period of the source that initiated the wave, and *not* on the properties of the medium. Once the wave period is set by the source, then all particles in the medium undergo the same period as well, as the wave propagates through the medium. This is due to the fact that the period is an **independent wave parameter**. (See **displacement, frequency, harmonic wave, independent wave parameters, medium, waves (water).**)



pressure/density wave ("sound")

(See **waves (pressure/density).**)

polarization

(See **displacement.**)

pulse wave

A **wave** whose y displacements at various x locations at t times is given by a general *pulse wave function*:

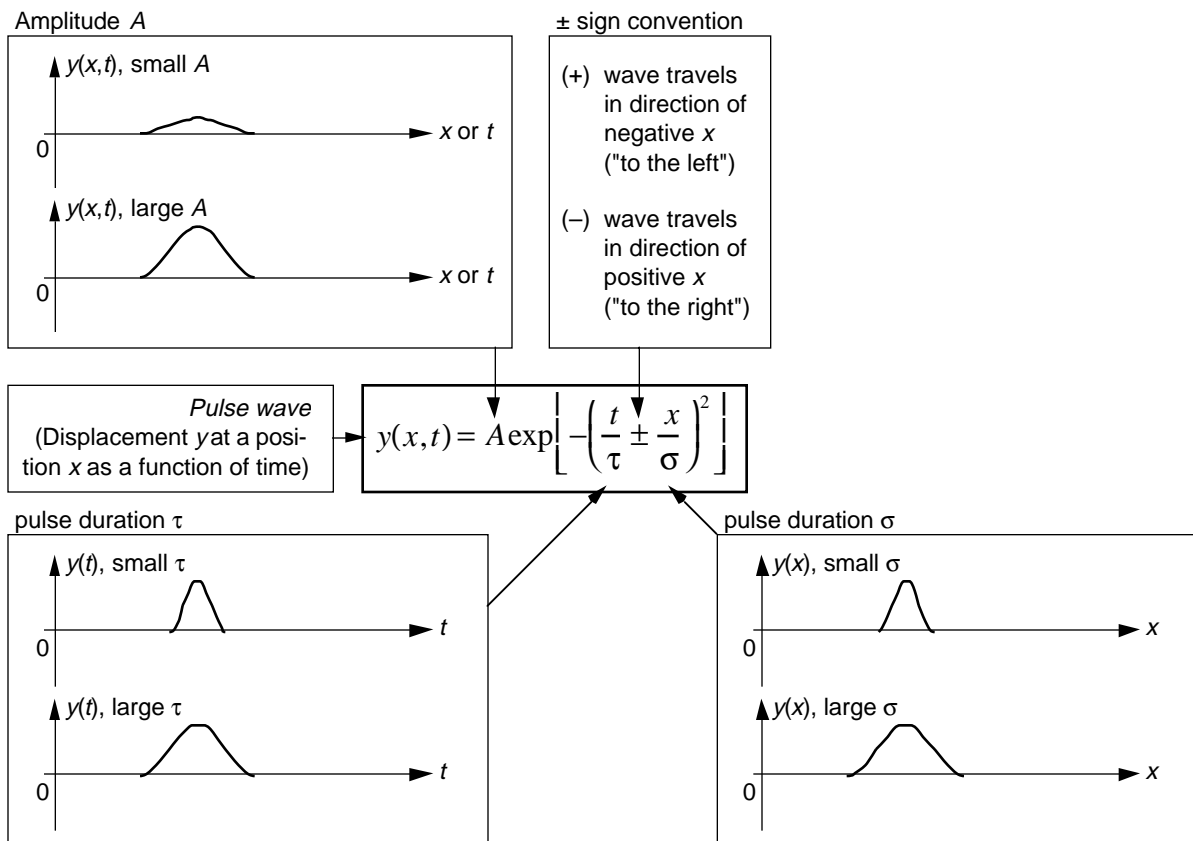
$$y(x,t) = A \exp\left[-\left(\frac{t}{\tau} \pm \frac{x}{\sigma}\right)^2\right].$$

A pulse wave is created by a singular disturbance—this is why we approximate a pulse wave with a Gaussian "hump."

The following parameters make up the pulse wave function, and are defined elsewhere in this glossary:

- **displacement** y .
- **amplitude** A .
- *pulse duration* τ .
- **location** x .
- *pulse spread* σ .

The (\pm) convention for wave direction works the same way for **harmonic waves**. This is because the maximum value for a pulse wave is when the exponent is zero! For the ($-$) choice, as time goes on (t becomes a bigger positive number), to "ride" the pulse, x must become a bigger positive number as well (such that the t term that gets bigger with time is canceled by the x term which also gets bigger with time), thus the wave moves in the positive x -direction (to the right). For the ($+$) choice, as time goes on (t becomes a bigger



positive number), to "ride" the pulse, x must become a bigger negative number (such that the t term is canceled by the x term), thus the wave moves in the negative x -direction (to the left).

The dependence of the pulse wave function on these parameters is schematically depicted on the previous page. (See **amplitude, displacement, harmonic wave, location, waves (general).**)

rope waves

(See **waves (rope).**)

"snapshot"

As a **harmonic wave** or a **pulse wave** travels through a **medium** (given by a $y(x,t)$ wave function), the particles in the medium undergoes a unique pattern of motion described by the function $y(x)$ that is "collapsed" from the original $y(x,t)$ wave function.

For a harmonic wave moving to the *left*, the $y(x,t)$ wave function is given by:

$$y(x,t) = A \sin\left(2\pi \frac{t}{T} + 2\pi \frac{x}{\lambda} + \psi_{\text{wave}}\right).$$

The $y(x)$ "snapshot" of all the particles at this specific time t_0 in the medium is found by substituting in the value $t = t_0$ into the $y(x,t)$ equation:

$$\begin{aligned} y(x,t_0) &= y(x,t)|_{t=t_0}, \\ &= A \sin\left(2\pi \frac{t_0}{T} + 2\pi \frac{x}{\lambda} + \psi_{\text{wave}}\right) \\ &= A \sin\left(2\pi \frac{x}{\lambda} + \underbrace{2\pi \frac{t_0}{T} + \psi_{\text{wave}}}_{\psi^{\text{snapshot}}}\right) \\ &= A \sin\left(2\pi \frac{x}{\lambda} + \psi^{\text{snapshot at } t=t_0}\right) \end{aligned}$$

If this is done for *each* and *every* time t , then you will get a sequence of $y(x)$ "snapshots" that make a "movie" of the wave moving through the particles in the medium.

This "collapse" of a $y(x,t)$ wave function to the $y(x)$ motion for the particles in a medium can be similarly done for pulse waves as well. (See **harmonic wave, location, medium, pulse wave, "snapshot."**)

"sound"*(See waves (pressure/density).)***total phase angle Ψ**

Everything inside the parenthesis in the argument of the sine function in the **harmonic wave** function. The value of the total phase angle $\Psi(x,t)$ depends, of course, on both the time t and position x along the wave (in contrast to the **constant phase angle** ψ_{wave} , which really is constant):

$$\Psi(x,t) = \left(2\pi \frac{t}{T} \pm 2\pi \frac{x}{\lambda} + \psi_{wave} \right).$$

If we use Ψ to represent all that junk inside the parenthesis, the harmonic wave function then takes the very simple-looking form:

$$y(x,t) = A \sin(\Psi).$$

Again note that the total phase angle Ψ is represented by an upper-case Greek "psi," while the constant phase angle ψ_{wave} is represented by a lower-case Greek "psi." (*See constant phase angle, harmonic wave.*)

water waves*(See waves (water).)***wave velocity v_{wave}**

A **wave** can only travel at one constant speed through a given **medium**, and that this value is set by the properties of the medium. Each and every medium will have its own specific wave velocity! Ideally, for *all* waves, the wave velocity v_{wave} is proportional to a square root of a *restoring* parameter, and inversely proportional to the square root of an *inertial* parameter:

$$v_{wave} \propto \sqrt{\frac{\text{restoring parameter}}{\text{inertial parameter}}}.$$

For a one-dimensional **harmonic wave** or a **pulse wave**, the wave can only travel along either the $+x$ or $-x$ direction. (*See harmonic wave, independent wave parameters, medium, pulse wave, waves (general).*)

waves (general)

The energy transfers (as seen as a "moving **displacement**," whether **harmonic** or **pulse**) that travel through a **medium**. In a wave, the particles (or whatever) of the medium oscillate about their **equilibrium** positions (or values) in a particular kind of spatially and temporally "organized" way, as the wave (*i.e.*, energy) travels through the medium.

Keep in mind that a wave is *not* an actual propagation of "something" material—a wave is the transfer of *energy*, and the presence of this energy transfer is seen in a **displacement** from equilibrium that seems to "travel" through the medium.

For the purposes of Block 11 in Physics 7C, we will describe four types of wave phenomena as harmonic and/or pulse waves:

- rope waves
- "light" (TEM waves)
- "sound" (pressure/density waves)
- water waves.

(See **displacement**, **equilibrium**, **harmonic wave**, **pulse wave**, **waves (pressure/density)**, **waves (rope)**, **waves (TEM)**, **waves (water)**.)

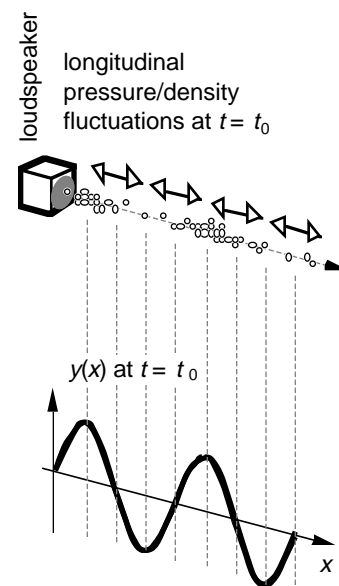
waves, pressure/density ("sound")

"Sound" is the colloquial term given to a wave (**harmonic** or **pulse**) that displaces the pressure or positions of atoms/molecules from their equilibrium values/positions, while propagates through a gaseous, liquid, or solid **medium**. This turns out to be a rather broad definition; as a result many different kinds of inaudible phenomena can be described as sounds.

The various wave parameters for harmonic sound waves in gases or liquids are given by:

- **displacement** $y =$ gauge pressure [Pa].
- **amplitude** $A =$ maximum gauge pressure fluctuation [Pa].
- **period** $T =$ repeat time of source (speaker) [s].
- **location** $x =$ location in medium [m].
- **wavelength** $\lambda =$ repeat distance in medium [m].
- **constant phase** $\psi_{wave} =$ source phase at $x = 0, t = 0$.

As with *all* idealized waves, the speed of sound waves depends solely on the properties of the medium, and is proportional to square root of its restoring properties (the incompressibility of gases, or interatomic/molecular forces in liquids/solids, which is given by the *bulk modulus* B), and inversely proportional to the square root of the density ρ of the medium:



$$v_{\text{wave, sound}} = \sqrt{\frac{B}{\rho}}.$$

In gases, the speed of sound is slightly less than the average speed of the molecules, and is temperature-dependent. For air at STP, we will be using a nominal value of $v_{\text{wave, sound}} = 340$ m/s. In general, sound travels nearly an order of magnitude faster in liquids and solids (1,000+ m/s) than through gases at room temperature and pressure, as the restoring forces in these phases are much greater than for gases, despite having a greater density.

In gases and liquids, sound waves can only have *longitudinal polarization displacements*. There are no sideways restoring forces (as for **rope waves**), so *transverse polarization displacements* are forbidden. In solids, sound waves can have either longitudinal or transverse polarizations, with different speeds for each type of polarization.

Keep in mind that the particles in the medium don't really go anywhere! They just bump back and forth in more or less the same position as the sound wave travels through them. For a sound wave in a liquid or solid, think of a crowded room where everyone is nearly touching each other—if people suddenly begin to crowd and push each other at one end of the room, then this "crowding" will travel across the room, although no person actually travels across the room at this speed.

Note that pressure displacements are not the same thing as equilibrium displacement (density) displacements. Strictly speaking the harmonic wave equations for pressure displacements have a constant phase that has a $\pi/2$ (90°) difference between their harmonic wave equations, but for the purposes of Block 11 in Physics 7C, we will not distinguish between the different constant phases for pressure and density displacement equations. For now, concentrate on the dependent and independent wave parameters of light waves in the **total phase angle** of its **harmonic wave** equation.

As with *all* waves, sound waves have frequencies determined by the source of the waves (typically longitudinally vibrating surfaces that rapidly compress/rarefy the pressure of the medium). The human ear is biomechanically engineered to respond only to sound frequencies in the range of 20 Hz to 20,000 Hz (as with all subjective experiences, this range will be slightly different for different people). Within this range, a single frequency sound is subjectively experienced as *pitch*—whether this pitch is a "musical" *note* or *tone* depends upon cultural musical conditioning and exposure. In Western cultures, most musical notes correspond to the black and white keys on a piano, even though there are certainly a vast number

of frequencies other than the frequencies of piano keys. For comparison, the range of sound frequencies a piano can produce is only 27.5 Hz to 4,186 Hz. There are certainly sounds that have frequencies lower or higher than a piano, or sounds that have frequencies lower or higher our range of perception. Sound frequencies lower than 20 Hz (called *subsonic*) are usually interpreted as "noise" rather than a tone as they are "felt" rather than heard. Sound frequencies higher than 20,000 Hz (called *hypersonic*) cannot be detected by humans at all (although there have been some clinically documented exceptions).

However, other animals (in particular, dogs and cats), can perceive sound frequencies higher than 20,000 Hz. Dog whistles produce hypersonic frequency sounds—we can't hear these sounds, but they are certainly audible to any dog. Newborn kittens typically meow at hypersonic frequencies, so while they may seem to us to be meowing silently, they are in fact easily heard by their mother cat. When these kittens mature, the frequency of their meowing sounds begins to lower into the human range of hearing. Early interaction with people will accelerate this process, as kittens soon learn that hypersonic meows are ignored by humans. Why is our frequency range of hearing seemingly so limited in comparison to other animals? One would suppose that this range is the evolutionary result of being able to hear edible prey. Cats and dogs use their hypersonic sense of hearing to find rodents and other smaller animals, which communicate using those frequencies.

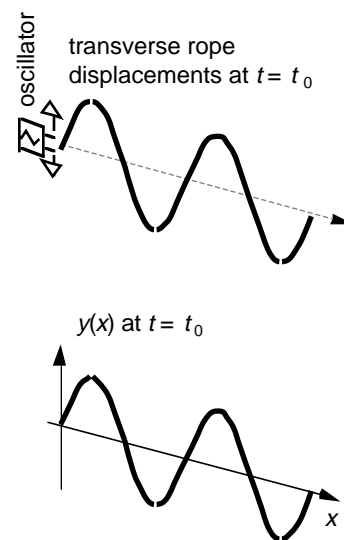
The subjective experience of sound wave amplitude is *loudness*, and can be approximate with logarithmic units of *decibels* (dB). As with all subjective experiences, pitch and loudness sensations vary immensely from person to person (and even moment-to-moment)! (See **displacement**, **harmonic wave**, **medium**, **pulse wave**, **waves (rope)**, **waves (general)**.)

waves, rope

The wave (**harmonic** or **pulse**) that displaces the transverse positions of the particles in a rope (string, cable, *etc.*) from their equilibrium positions.

The various wave parameters for harmonic rope waves are given by:

- **displacement** y = transverse position [m].
- **amplitude** A = maximum transverse fluctuation [m].
- **period** T = repeat time of source (hand or oscillator) [s].
- **location** x = location along rope [m].
- **wavelength** λ = repeat distance along rope [m].
- **constant phase** ψ_{wave} = source phase at $x = 0$, $t = 0$.



As with *all* ideal waves, the speed of rope waves depends solely on the properties of the **medium**, and is proportional to square root of its restoring properties (the tension force $F_{tension}$ along the rope), and inversely proportional to the square root of the "linear density" μ of the rope (which is its total mass, in kg, divided by its total length, in m):

$$v_{wave, rope} = \sqrt{\frac{F_{tension}}{\mu}}.$$

Thus transverse displacement waves travel fastest through taut and very thin ropes, string, and cables.

For ropes, waves can only have *transverse polarization displacements*. There are no back-to-front restoring forces (as in a Slinky™), so *longitudinal polarization* displacements are forbidden (you can't "push" a rope like you can a Slinky™!).

As with *all* waves, rope waves have frequencies determined by the source of the waves (typically something longitudinally disturbing the rope). (*See amplitude, displacement, harmonic wave, medium, pulse wave, waves (general).*)

waves, TEM ("light")

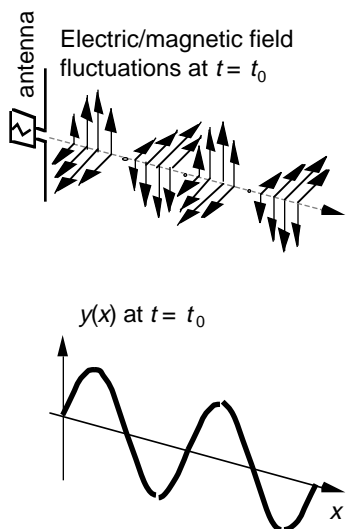
"Light" is the colloquial term given to a *Transverse Electromagnetic Wave* (**harmonic** or **pulse**) that displaces the *electric* and *magnetic fields* from their equilibrium values, while it propagates through a **medium**.

The various wave parameters for harmonic light waves are given by:

- **displacement** y = electric or magnetic field [N/Coul or Teslas].
- **amplitude** A = maximum electric or magnetic field fluctuation [N/Coul or Teslas].
- **period** T = repeat time of source (antenna) [s].
- **location** x = location in medium [m].
- **wavelength** λ = repeat distance in medium [m].
- **constant phase** ψ_{wave} = source phase at $x = 0, t = 0$.

As with *all* ideal waves, the speed of light waves depends solely on the properties of the medium. Instead of the generalized "square-root restoring/inertial" form, light wave speeds are usually given as an expression of *index of refraction* n_{medium} which is a unitless quantity that depends solely on the properties of the medium:

$$v_{wave, light} = \frac{c}{n_{medium}} = \frac{(3.00 \times 10^8 \text{ m/s})}{n_{medium}}.$$



Typically light waves travel slower through denser (transparent) media, and quicker through less dense media, and travels at its maximum speed of $c \equiv 3.00 \times 10^8$ m/s through a vacuum. Thus $n_{\text{medium}} = 1$ for a vacuum, and is approximately correlated with the density of a transparent medium, such that n_{medium} is slightly greater than unity for air at STP, and $n_{\text{medium}} > 1$ for all other transparent materials.

Light waves can only have *transverse polarization displacements*—hence the formal term "TEM waves." (For the purposes of Block 11 in Physics 7C, it is not important to know *what* electric and magnetic fields are, which are covered in Block 13 of Physics 7C. For now, concentrate on the dependent and independent wave parameters of light waves in the **total phase angle** of its **harmonic wave** equation.)

As with *all* waves, light waves have frequencies determined by the source of the waves (usually varying the current of electrons in wires, and/or the vibrations of atoms/molecules). The human eye is biochemically engineered to respond only to light frequencies in the range of 4.3×10^{14} Hz to 7.5×10^{14} Hz (as with all subjective experiences, this range will be slightly different for different people). Within this range, light is subjectively experienced as *color*—red corresponds to the lower frequency limit; violet corresponds to the upper frequency limit; and all other colors correspond to frequencies *within* this *visible light* range. Light frequencies just lower than 4.3×10^{14} Hz are called *infrared* (*i.e.*, "lower than red"), and even lower frequencies correspond to *microwaves*, then TV, FM, and AM *radio* waves. Light frequencies just higher than 7.5×10^{14} Hz are called *ultraviolet* ("beyond violet"), and even higher frequencies correspond to *x-rays* and *gamma rays*. Taken together, *all* of these waves are referred to as light. "Visible" light occupies one small part of the total frequency range of light waves. As with sound waves and our ears, there are quite a lot of different types of light frequencies that cannot be perceived by our eyes. Just because we cannot detect and subjectively perceive these different frequencies of light as color does not mean that they do not exist—we only need to construct different types of "detectors" (like the microwave detectors used in DL) that are sensitive to these other light frequencies in order to "see" them.

An interesting effect of trying to experience many different visible light frequencies simultaneously is that many different colors combine to form "white" light. Thus white is really not a color (*i.e.*, not a single frequency light wave), but the combination of many different visible frequency light waves.

The subjective experience of light wave intensity is *brightness*. As with all subjective experiences, color and brightness sensations vary immensely from person to person (and even moment-to-

moment)! (See **displacement**, **harmonic wave**, **medium**, **pressure/density wave ("sound")**, **pulse wave**, **waves**.)

waves (water)

The wave (**harmonic** or **pulse**) that displaces the positions of the surface of a liquid from their equilibrium positions.

The various wave parameters for harmonic water waves are given by:

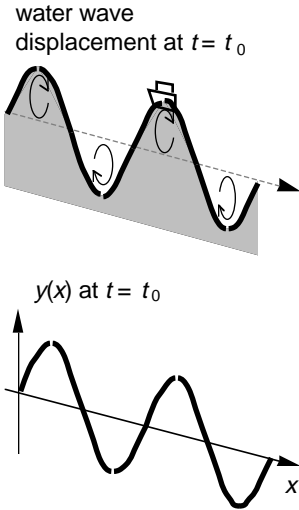
- **displacement** y = displacement [m].
- **amplitude** A = maximum displacement ("wave height") [m].
- **period** T = repeat time of source [s].
- **location** x = location in medium [m].
- **wavelength** λ = repeat distance in medium [m].
- **constant phase** ψ_{wave} = source phase at $x = 0, t = 0$.

There is an important word of caution when discussing water waves—despite their appeal as the standard model of waves in general, water waves they are not ideally described in terms of the wave models and concepts developed in Block 11 in Physics 7C, and thus can only be crudely approximated as transverse harmonic waves. First, note that water waves have *both transverse and longitudinal displacements*, and are approximately circular in nature (think of floating on an inner tube far offshore on a lake, or in the ocean, with smooth rolling waves—you will move backwards into the wave trough, then get carried upwards, move forwards along with the wave crest, and then move downwards as the wave crest passes—don't get seasick!). For the purposes of Block 11 in Physics 7C, we will make the crude approximation that water waves are exclusively transversely polarized!

Ideally for *all* waves, the speed of waves depends solely on the properties of the **medium**. However, the speed of water waves not only depends on the properties of the medium (the **gravitational field constant** g , and the depth of the water d); the speed of water waves *also* depends on the **wavelength** of the waves!

$$v_{wave, water} = \sqrt{\frac{g\lambda}{2\pi} \tanh\left(2\pi \frac{d}{\lambda}\right)}.$$

Think of floating on an inner tube far offshore on a lake, or in the ocean, with smooth rolling waves (no breakers). The large wavelength waves will move very quickly through your position. If you dip your finger into the water to make small wavelength ripples, these small wavelength waves will move with a much different speed than the larger wavelength rolling waves.



The "tanh" function is the "hyperbolic tangent" (pronounced "tansh"), and for the purposes of Block 11 in Physics 7C, it suffices to know that $\tanh(u)$ takes the limits of $\tanh(0) = 0$, and $\tanh(\infty) = 1$. Thus for very deep water ($d \rightarrow \infty$), the speed of water waves is given by:

$$v_{\text{wave, water}} \approx \sqrt{\frac{g\lambda}{2\pi}},$$

which is *still* wavelength-dependent and non-ideal. Note that for very shallow water ($d \rightarrow 0$), the speed of water waves approaches zero. You may have noticed this when standing at the shore of a sandy beach, where the breakers approach the shore very quickly (run away!), and then as they wash up onto the shore they slow down dramatically, and you can actually walk away from the very edge of the waves that wash up onto the sand.

As with *all* waves, water waves have frequencies determined by the source of the waves (typically something disturbing the water, such as your finger in the example given above, or maybe a very large wave maker at water amusement parks). However, very large wavelength waves are typically produced by drag forces created by winds continuously blowing across the surface of the water, and thus these types of wave don't necessarily have to have a strictly periodic source.

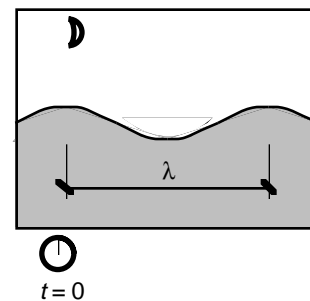
Thus for the purposes of Block 11 in Physics 7C, we will often refer to water waves as being described by harmonic transverse waves, but keep in mind that this is strictly a convenient idealization. (*See **displacement, harmonic wave, medium, pulse wave, waves (general).***)

wavelength λ

This is the repeat distance (in m) that you can measure in a **medium**, along the x direction for a **harmonic wave** to repeat itself. If you are still thinking of **water waves**, the wavelength is the horizontal distance from one crest to the next crest; which is the same as the horizontal distance from one trough to the next trough, *etc.*

Wavelength is a **dependent wave parameter** that depends on both the properties of the source (**period T**), and the properties of the medium (**wave velocity v_{wave}**):

$$\lambda = \frac{v_{\text{wave}}}{f} = v_{\text{wave}} T.$$



(See **dependent wave parameter, harmonic wave, medium, period, wave velocity, waves (water).**)

Block 12 Glossary

Wave superposition glossary

constant phase angle ψ_{wave}	path-length L
constructive interference	path-length difference ΔL
destructive interference	reflection phase shift $\psi_{reflection}$
impedance	source phase angle ψ_{source}
"in-phase"	total phase angle Ψ
index of refraction n	total phase difference $\Delta\Psi$
interference conditions	wave superposition
"out-of-phase"	wave velocity v_{wave}

constant phase angle ψ_{wave}

The constant phase angle ψ_{wave} of a harmonic wave (first considered in Block 11 of Physics 7C) is actually made up of two other constant phase angles—the **source phase angle** ψ_{source} , and the **reflection phase shift** $\psi_{reflection}$:

$$\psi_{wave} = \psi_{source} + \psi_{reflection}.$$

(See **reflection phase shift**, **source phase angle**.)

constructive interference

Consider two harmonic waves of the same amplitude that are superposed. The two waves are said to interfere *constructively* if their **superposition** results in a wave that has *twice* the amplitude as before. In this case the two waves that are superposed that are said to be "**in-phase**" with each other. At right is an example of two "in-phase" harmonic waves of the same period/wavelength that constructively interfere with each other.

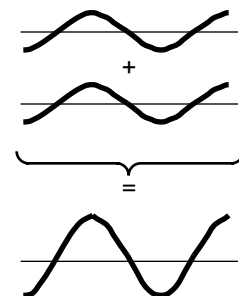
Two harmonic waves of *any* given period/wavelength will interfere constructively (regardless of the relative in/out phases of the sources and/or reflections involved) if their **total phase difference** satisfies the following **interference condition**:

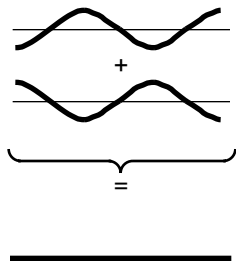
$$\Delta\Psi = \pm(\text{even})\pi.$$

(See "**in-phase**," **interference conditions**, **total phase difference**.)

destructive interference

Consider two harmonic waves of the same amplitude that are superposed. The two waves are said to interfere *destructively* if their **superposition** results in a wave that has *zero* amplitude. In this case the two waves that are superposed are said to be "**out-of-**





phase" with each other. At left is an example of two "out-of-phase" harmonic waves of the same period/wavelength that destructively interfere with each other.

Two harmonic waves of *any* given period/wavelength will interfere destructively (regardless of the relative in/out phases of the sources and/or reflections involved) if their **total phase difference** satisfies the following **interference condition**:

$$\Delta\Psi = \pm(\text{odd})\pi.$$

(See "in-phase," **interference conditions, total phase difference**.)

impedance

A measure of the "difficulty" a wave has in propagating through a medium. The impedance of a medium is inversely correlated with the velocity of waves through that medium.

For light waves, the impedance of a medium is correlated with the valence electrons within that medium, and is *proportional* to density (and the **index of refraction**) of that medium.

For sound waves, the impedance of a medium is *inversely proportional* to the density of a medium, as dense materials typically have *stronger* interatomic/molecular restoring forces that result in faster sound wave velocities.

The relative values of the impedance of a reflective surface, and the medium above it (where the incident and reflected waves are) determine the **reflection phase shift** of that wave, after bouncing back off of that medium. (See **index of refraction, reflection phase shift**.)

"in-phase"

An informal term used to describe any situation where *something* about two harmonic waves is coinciding with each other.

If the simple harmonic motion of the sources of two harmonic waves are synchronized (*i.e.*, they both have the same **source phase angle** ψ_{source}), then the *sources* of these waves are said to be "in-phase" with each other.

If the two harmonic waves both reflect off of surfaces with similar impedance differences (*i.e.*, they both have the same **reflection phase shift** $\psi_{\text{reflection}}$), then the *reflections* of these waves are said to be "in-phase" with each other.

If the superposition of these harmonic waves results in **constructive interference**, then these *waves* are said to be "in-phase" with each other. (See **constructive interference, reflection phase shift, source phase angle**.)

index of refraction n

A unitless ratio n measuring "optical density," or how slow light waves travel through a transparent medium, compared to the velocity of light waves in vacuum:

$$n_{\text{medium}} = \frac{c}{v_{\text{medium}}},$$

where $c \equiv 3.00 \times 10^8$ m/s is the velocity of light waves in that medium, and v_{material} is the (slower) velocity of light waves in the transparent medium. Light waves always travel slower through all other media, compared to traveling through a vacuum.

Roughly speaking, the index of refraction of a medium is correlated with the **impedance** of that medium. The index of refraction of a medium is somewhat correlated with the density of that medium.

Indices of refraction for several common transparent media are listed at right. Note that because the velocity of light is slower in all transparent media than through a vacuum, all indices of refraction are greater than 1.

E.g., water has an index of refraction of $n = 1.329$, which means that light waves travel 1.329 times *slower* through water than through vacuum. The velocity of light waves through water is given by:

$$v_{\text{water}} = \frac{c}{n_{\text{water}}} = \frac{3.00 \times 10^8 \frac{\text{m}}{\text{s}}}{1.329} = 2.257 \times 10^8 \frac{\text{m}}{\text{s}}.$$

(See **impedance**.)

Media	Index of refraction n
vacuum	1
air	1.00029
ice	1.31
water	1.329
fused quartz	1.4584
benzene	1.501
Plexiglas™	1.51
crown glass	1.52
zircon	1.923
diamond	2.417

interference conditions

Whether the **superposition** (*adding*) of two harmonic waves will result in either **constructive interference** or **destructive interference** can be determined by *subtracting* certain wave parameters from each other.

Harmonic waves emitted from two **in-phase** sources with the same wavelength will interfere constructive or destructively depending on their **path-length difference ΔL** :

$$\Delta L = \begin{cases} (\#)\lambda & \text{constructive,} \\ (\# + \frac{1}{2})\lambda & \text{destructive.} \end{cases}$$

The above interference condition will also be true for harmonic waves emitted from two **out-of-phase** sources with the same wavelength, and with out-of-phase **reflection phase shifts**.

Harmonic waves emitted from two out-of-phase sources with the same wavelength will interfere constructive or destructively depending on their path-length difference ΔL :

$$\Delta L = \begin{cases} (\# + \frac{1}{2})\lambda & \text{constructive,} \\ (\#)\lambda & \text{destructive.} \end{cases}$$

The above interference condition will also be true for harmonic waves emitted from two in-phase sources with the same wavelength, and with out-of-phase reflection phase shifts.

Most generally the interference condition for *any* type of sources (in-phase or out-of-phase) and *any* type of reflection phase shift (in-phase or out-of-phase) can be summarized with just *one* equation involving the **total phase difference** $\Delta\Psi$:

$$\Delta\Psi = \left(-2\pi \frac{\Delta L}{\lambda} + \Delta\Psi_{\text{sources}} + \Delta\Psi_{\text{reflections}} \right) = \begin{cases} \pm(\text{even})\pi & \text{constructive,} \\ \pm(\text{odd})\pi & \text{destructive.} \end{cases}$$

This is the most universal form of determining whether two harmonic waves of the *same* wavelength will interfere constructively or destructively, regardless of the relative in/out phases of the sources and/or reflections involved.

However, what about the superposition of two waves that have *different* wavelengths? It turns out that the interference condition can still be expressed in terms of the total phase difference, but will now be time-dependent:

$$\Delta\Psi = \left(2\pi t(\Delta f) - 2\pi\Delta\left(\frac{L}{\lambda}\right) + \Delta\Psi_{\text{sources}} + \Delta\Psi_{\text{reflections}} \right) = \begin{cases} \pm(\text{even})\pi & \text{constructive,} \\ \pm(\text{odd})\pi & \text{destructive.} \end{cases}$$

This is the most universal form of determining whether two harmonic waves of *any* wavelength will interfere constructively or destructively, regardless of the relative in/out phases of the sources and/or reflections involved. (See **constructive interference, destructive interference, "in-phase," "out-of-phase," path-length difference, reflection phase shift, source phase angle, total phase difference.**)

"out-of-phase"

An informal term used to describe any situation where *something* about two harmonic waves is *not* coinciding with each other.

If the simple harmonic motion of the sources of two harmonic waves are completely out-of-synchronization (*i.e.*, they have

constant phase angles that differ by π), then the *sources* of these waves are said to be "out-of-phase" with each other.

If the two harmonic waves both reflect off of surfaces with different impedance differences (*i.e.*, they have **reflection phase shifts** that differ by π), then the *reflections* of these waves are said to be "out-of-phase" with each other.

If the superposition of these harmonic waves results in **destructive interference**, then these *waves* are said to be "out-of-phase" with each other. (See **destructive interference, reflection phase shift, source phase angle.**)

path-length L

The linear "elapsed" distance a wave travels, in spite of any reflections (which result in reversals of direction). For example, consider a *harmonic wave* starts from the $x = 0.0$ m origin, and travels to the left to a reflective barrier at $x = -1.0$ m, and then travels back to the right. When this wave reaches the origin again, it will have traveled a path-length $L = 2.0$ m. Thus path-lengths are *always* positive definite quantities, as the L for a wave is always measured "outwards" from its source along whatever path it is forced to take. (So be warned: x and L are not interchangeable, but are related. There is no unique mathematical relation between location x and path-length L for all the situations to be considered in Block 12 of Physics 7C, as L is usually determined from inspection of the specific case being considered.)

Because of this, if we express a harmonic wave in terms of path-length L instead of location x , there is *always* a $(-)$ sign in the **total phase angle**:

$$\begin{aligned} y(L, t) &= A \sin(\Psi), \\ &= A \sin\left(2\pi \frac{t}{T} - 2\pi \frac{L}{\lambda} + \Psi_{\text{wave}}\right) \\ &= A \sin\left[2\pi \frac{t}{T} - 2\pi \frac{L}{\lambda} + (\Psi_{\text{source}} + \Psi_{\text{reflection}})\right]. \end{aligned}$$

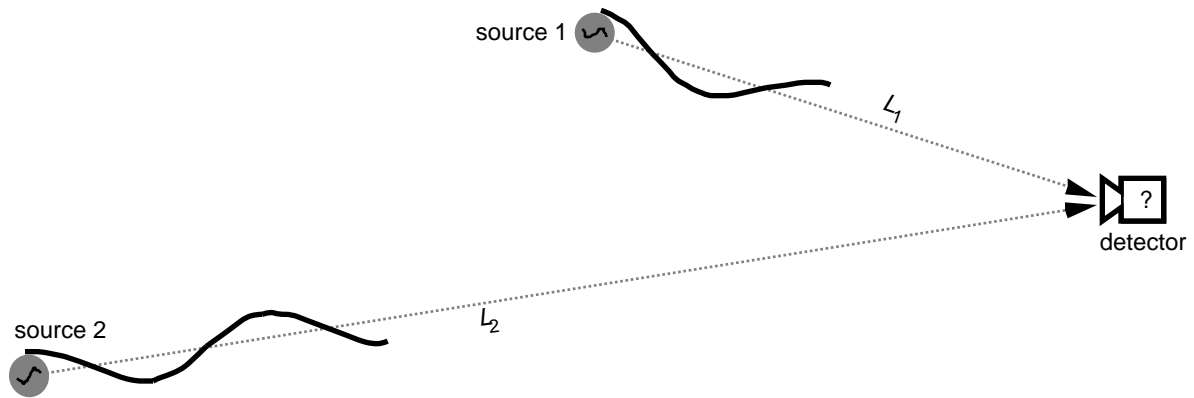
(See **total phase angle.**)

path-length difference ΔL

How much further one wave travels from its source, to a specific location, than another wave that travels from its source, to that same location. This is just the algebraic subtraction of the **path-lengths** of the two harmonic waves:

$$\Delta L = L_1 - L_2.$$

Note that this is not the change in path-length of one given harmonic wave (i.e., final path-length minus initial path-length); this is merely the subtraction of the path-length of one wave from the path-length of another wave, and to emphasize this, we will deliberately subtract them the "wrong way" ("1 minus 2").



The path-length difference between two **superposed** harmonic waves can determine whether they will result in **constructive interference** or **destructive interference**, as ΔL appears in several different **interference condition** forms. (See **constructive interference, destructive interference, interference conditions, path length difference, wave superposition.**)

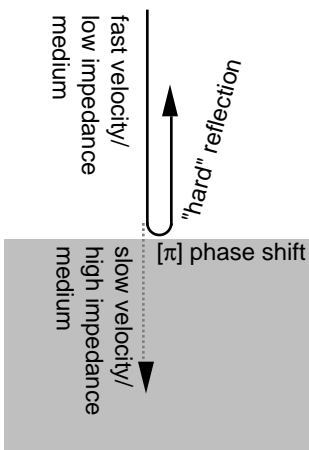
reflection phase shift $\psi_{\text{reflection}}$

A harmonic wave will "shift" its **constant phase angle** ψ_{wave} if it reflects off of an interface between media with different impedances. Note that the constant phase angle is actually comprised of the **source phase angle** ψ_{source} , which takes care of what the original wave source was doing at $t = 0$, along with an explicit contribution from reflections:

$$\psi_{\text{wave}} = \psi_{\text{source}} + \psi_{\text{reflection}}$$

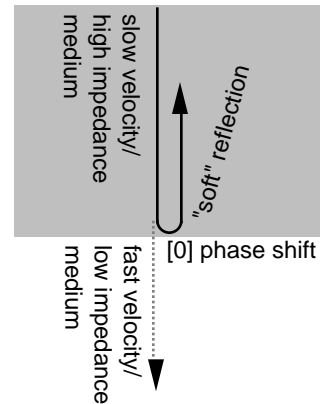
If a harmonic wave does not reflect off of anything, then its reflection phase shift $\psi_{\text{reflection}} \equiv 0$. However, $\psi_{\text{reflection}}$ may or may not be zero once the wave bounces backwards off of something, depending on the relative **impedance** values:

- If a harmonic wave in a *low* impedance medium reflects off of a *high* impedance medium, then $\psi_{\text{reflection}} = \pi$. (Equivalently, $\psi_{\text{reflection}} = \pi$ if the harmonic wave in a *fast* velocity medium reflects off of a *slow* velocity medium.)



- If a harmonic wave in a *high* impedance medium reflects off of a *low* impedance medium, then $\psi_{\text{reflection}} = 0$. (Equivalently, $\psi_{\text{reflection}} = 0$ if the harmonic wave in a *slow* velocity medium reflects off of a *fast* velocity medium.)

(See **constant phase angle, impedance, source phase angle.**)



source phase angle ψ_{source}

This is "where" the source of a harmonic wave "is" with respect to its periodic cycle at $t = 0$, and at $L = 0$. Previously in Block 11 of Physics 7C this was synonymous with the **constant phase angle ψ_{wave}** of a harmonic wave. However, because we are also considering the *reflection* of waves, the constant phase angle of a wave includes both the source phase angle and **reflection phase shifts**:

$$\psi_{\text{wave}} = \psi_{\text{source}} + \psi_{\text{reflection}}$$

(See **constant phase angle, reflection phase shift.**)

total phase angle Ψ

Everything inside the parentheses in the argument of the sine function in the *harmonic wave* function. The value of the total phase angle $\Psi(L, t)$ depends, of course, on both the time t and path-length L along the wave (in contrast to the **constant phase angle ψ_{wave}** , which really is constant):

$$\begin{aligned} \Psi(L, t) &= \left(2\pi \frac{t}{T} - 2\pi \frac{L}{\lambda} + \psi_{\text{wave}} \right) \\ &= \left[2\pi \frac{t}{T} - 2\pi \frac{L}{\lambda} + (\psi_{\text{source}} + \psi_{\text{reflection}}) \right]. \end{aligned}$$

If we use Ψ to represent all that junk inside the parenthesis, the harmonic wave function then takes the very simple-looking form:

$$y(L, t) = A \sin(\Psi).$$

Again note that the total phase angle Ψ is represented by an upper-case Greek "psi," while the constant phase angle ψ_{wave} is represented by a lower-case Greek "psi" (which is itself comprised of a **source phase angle ψ_{source}** and a **reflection phase shift $\psi_{\text{reflection}}$**). (See **constant phase angle, reflection phase shift, source phase angle.**)

total phase difference $\Delta\Psi$

The algebraic subtraction of the **total phase angles** of two harmonic waves:

$$\Delta\Psi = \Psi_1 - \Psi_2.$$

Note that this is not the change in total phase of one given harmonic wave (i.e., final total phase minus initial total phase); this is merely the subtraction of the total phase of one wave from the total phase of another wave, and to emphasize this, we will deliberately subtract them the "wrong way" ("1 minus 2").

Whether the **superposition** of two harmonic waves results in **constructive interference** or **destructive interference** depends merely on the **interference condition** where the difference between their total phase angles is either an even or an odd multiple of π :

$$\Delta\Psi = \begin{cases} \pm(\text{even})\pi & \text{constructive,} \\ \pm(\text{odd})\pi & \text{destructive.} \end{cases}$$

(See **constructive interference, destructive interference, interference conditions, total phase angle, wave superposition.**)

wave superposition

Consider two waves whose harmonic wave functions are given by $y_1(x,t)$ and $y_2(x,t)$. Wave superposition is nothing more than the addition of these two waves together, resulting in a total wave $y_{total}(x,t)$:

$$y_{total} = y_1 + y_2.$$

Mathematically this means that we are adding together the values of two waves, at each and every different location x , and for all subsequent values of time t . For the purposes of Block 12 in Physics 7C, we will consider the results of superposing only *two* waves together. When two waves are superposed, we observe the effects of *wave interference*.⁵

There are *two* methods to determine the type of interference (**constructive interference** or **destructive interference**) resulting from superposing two waves together:

⁵ What is the difference between *superposition* and *interference*? Superposition is the general principle of adding quantities. Interference is specifically the adding (superimposing) of two (or more) waves. For the purposes of Block 12 in Physics 7C these two terms will be synonymous.

- Literally *adding* the two waves together, mathematically, and see what happens.
- *Subtracting* the total phase of one wave from the total phase of the other wave to determine the **total phase difference** $\Delta\Psi$, and make predictions on what happens *if* the two waves were added together, by looking at the **interference conditions**.

(See **constructive interference, destructive interference, interference conditions, total phase difference.**)

wave velocity v_{wave}

Wave superposition only concerns the addition of two waves of the same type (whether they are both sound waves, or both light waves, *etc.*). This means that the two waves that we are adding together also *must* be in the *same* medium (such that they can be in the same location at the same time), and thus they both *must* have the *same* wave velocity v_{wave} . Recall that the wavelength of a harmonic wave depends on both the period of the wave source, and the velocity of the wave through the medium. This places a very important restriction on the wavelengths and periods of the harmonic waves that can interfere with each other:

$$\left. \begin{array}{l} \lambda_1 = v_{\text{wave}} T_1 \\ \lambda_2 = v_{\text{wave}} T_2 \end{array} \right\} \rightarrow \frac{\lambda_1}{T_1} = \frac{\lambda_2}{T_2}.$$

There are *two* cases regarding wavelengths and periods of the harmonic waves that can interfere with each other:

- If the two waves we are going to add together have the *same* period, then since they *must* have the *same* velocity, they will have the *same* wavelength:

$$\frac{\lambda_1}{T_1} = \frac{\lambda_2}{T_2} \rightarrow \left\{ \begin{array}{l} T_1 = T_2, \\ \lambda_1 = \lambda_2. \end{array} \right.$$

- If the two waves we are going to add together have *different* periods (certainly allowable!), then since they *must* have the *same* velocity, they will have *different* wavelengths.

$$\frac{\lambda_1}{T_1} = \frac{\lambda_2}{T_2} \rightarrow \left\{ \begin{array}{l} T_1 \neq T_2, \\ \lambda_1 \neq \lambda_2. \end{array} \right.$$

(See **wave superposition.**)

Applications glossary

antinode

beats

beat frequency f_{beat}

beat period T_{beat}

carrier frequency $f_{carrier}$

carrier period $T_{carrier}$

"diffraction grating"

double-slit interference

fundamental frequency

"harmonic frequency"

node

resonance

standing wave

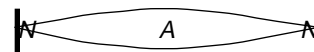
thin film interference

antinode

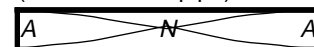
An *antinode* is the location where two oppositely traveling waves of the same period and same wavelength *always* experience **constructive interference**. Observations of antinodes depends on the specific type of **standing waves** involved:

- For a standing wave on a rope, every location where the rope has a maximal transverse excitation is an antinode.
- For a standing sound wave (in air), every location where the gauge pressure of the air undergoes a maximal fluctuation is said to be a pressure fluctuation antinode, as with a closed end of a pipe.⁶

Transverse fluctuations
(closed-closed rope)



Pressure fluctuations
(closed-closed pipe)



(See **constructive interference**, **node**, **standing wave**.)

beats

Consider two harmonic (sound) waves with *different periods* and *different wavelengths* that meet at a specific location and time, after both traveling their respective path-lengths from their respective sources. These two waves will experience either **constructive interference** or **destructive interference**, depending on the most general form of the **interference condition** given by their **total phase difference**:

$$\Delta\Psi = \left(2\pi t(\Delta f) - 2\pi\Delta\left(\frac{L}{\lambda}\right) + \Delta\Psi_{sources} + \Delta\Psi_{reflections} \right) = \begin{cases} \pm(\text{even})\pi & \text{constructive,} \\ \pm(\text{odd})\pi & \text{destructive.} \end{cases}$$

⁶ However, every location where the air molecules undergo a maximal displacement from their equilibrium positions is said to be a position displacement antinode. These are *not* the same locations as pressure fluctuation antinodes! This is because for a given sound wave, its pressure fluctuations and position displacements are shifted 90° from each other. For example, the closed end of a pipe is a pressure fluctuation antinode, as air will be compressed and uncompressed there. However, the closed end of a pipe is a position displacement **node**, as you cannot displace the air molecules immediately next to the wall! Also consider an open end of a pipe, which is a position displacement antinode, as the air molecules there can easily be displaced from their equilibrium positions. However, the open end of a pipe is a pressure fluctuation node, as the end open to the atmosphere must *always* remain at atmospheric pressure!

The key difference between this type of interference, which involves two waves of different periods/wavelengths, and with all of the other interference phenomena considered in Block 12 of Physics 7C is that this dependence is *time-dependent*. These two different period/wavelength waves will interfere constructively at one given time, then they will interfere destructively a short time later, then interfere constructively again, *etc.*

In order to focus on this time-dependent behavior, the non-time-dependent terms in the total phase difference can be grouped together into a single constant:

$$\Delta\Psi = \left(2\pi t \underbrace{(f_1 - f_2)}_{f_{beat}} + [constant] \right) = \begin{cases} \pm(\text{even})\pi & \text{constructive,} \\ \pm(\text{odd})\pi & \text{destructive.} \end{cases}$$

The **beat frequency** is the *difference* of the frequencies of the two sound waves:

$$f_{beat} = |f_1 - f_2|.$$

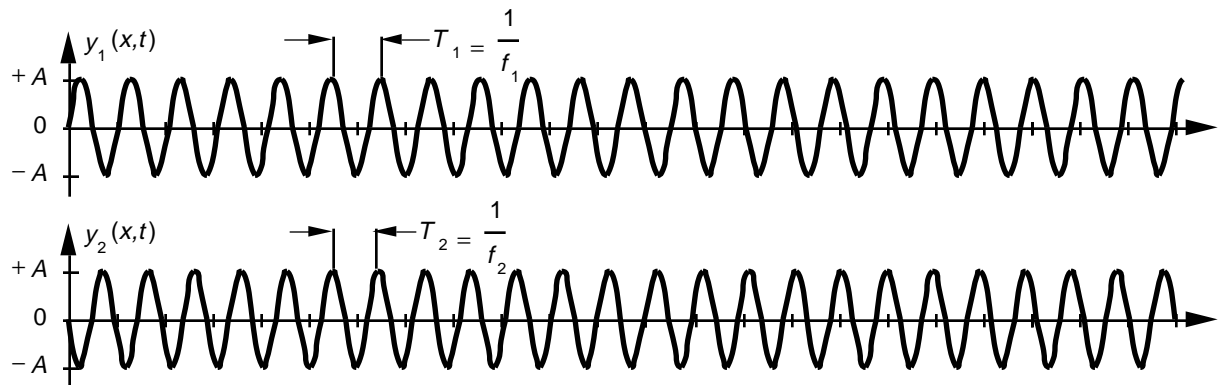
This is the frequency (in Hz) of the number of times per second the superposition of these two waves goes through a constructive-destructive-constructive cycle. If two sound waves of different periods/wavelengths are superposed, then you will hear a loud superposed sound (constructive interference), then zero sound (destructive interference), then a loud sound, *etc.* This can be described informally as "wobbling," and when musicians are adjusting their instruments to the same reference frequency (usually 440 Hz); a discernible wobbling means that their instruments are not yet "in tune" with each other.

The **carrier frequency** $f_{carrier}$ is the *average* of the frequencies of the superposition of two sound waves:

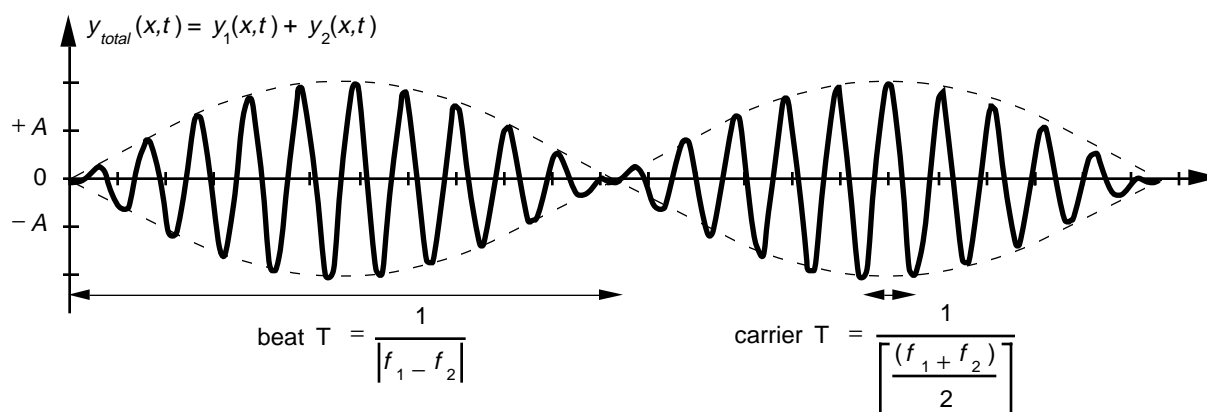
$$f_{carrier} = \frac{1}{2}(f_1 + f_2).$$

This is the frequency (in Hz) of the number of peaks (or the number of troughs) per second of the superposition of these two harmonic waves. If two sound waves of different periods/wavelengths are superposed, this is the perceived tone that gets "wobbled."

What does this look like, graphically? First consider the two waves with different periods (and thus different wavelengths):



Then the superposition of these two waves is merely their graphical addition.



Note that there are two distinct types of periodic behavior. The "big wiggly" is the beat period T_{beat} , which is the time it takes for the superposed wave to go through a complete constructive-destructive-constructive (loud-soft-loud) cycle. The "small wiggly" is the carrier period $T_{carrier}$, which is the time between consecutive peaks (or troughs) for all non-destructive times. (*See **beat frequency, beat period, carrier frequency, carrier period, constructive interference, destructive interference, interference conditions.***)

beat frequency f_{beat}

The beat frequency is the *difference* of the frequencies of the superposition of two harmonic waves with different periods and wavelengths:

$$f_{beat} = |f_1 - f_2|.$$

This is the frequency (in Hz) of the number of times per second the superposition of these two waves goes through a constructive-

destructive-constructive cycle (or a loud-soft-loud cycle). (See **beats**.)

beat period T_{beat}

The beat period is the *inverse* of the **beat frequency** of two harmonic waves with different periods and wavelengths:

$$T_{beat} = \frac{1}{f_{beat}}.$$

This is the time (in seconds) for the superposition of these two waves to go through a constructive-destructive-constructive cycle (or a loud-soft-loud cycle). (See **beats**.)

carrier frequency $f_{carrier}$

The carrier frequency is the *average* of the frequencies of the superposition of two harmonic waves with different periods and wavelengths:

$$f_{carrier} = \frac{1}{2}(f_1 + f_2).$$

This is the frequency (in Hz) of the number of peaks (or the number of troughs) per second of the superposition of these two harmonic waves. (See **beats**.)

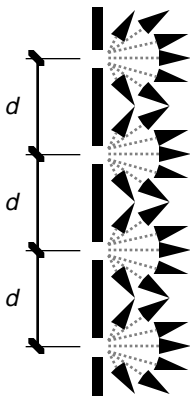
carrier period $T_{carrier}$

The carrier period is the *inverse* of the **carrier frequency** of two harmonic waves with different periods and wavelengths:

$$T_{carrier} = \frac{1}{f_{carrier}}.$$

This is the time (in seconds) between successive peaks (or troughs) of the superposition of these two harmonic waves. (See **beats**.)

"diffraction grating"



A diffraction grating is a barrier that has very tiny slits inscribed on it, all spaced the same distance d apart. A single wave incident on this system will result in a multitude of in-phase (same constant phase) sources whose waves will superpose either constructively or destructively along certain θ directions. These θ directions turn out to be the *same* θ directions that *two* in-phase waves from slits spaced an interval d apart will interfere constructively or destructively. Thus a diffraction grating behaves more or less like a double-slit system, because if two in-phase waves from slits spaced an interval d apart

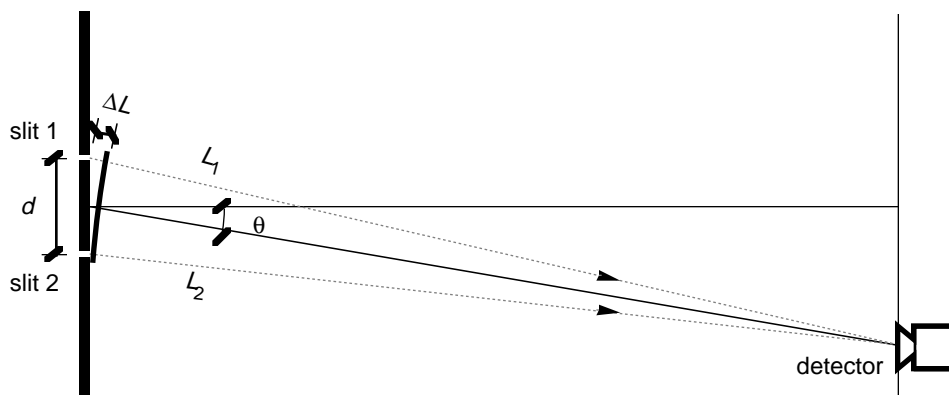
interfere constructively or destructively along a certain θ direction, then many, many in-phase waves from slits spaced an interval d apart interfere constructively or destructively along that *same* θ direction. (See **double-slit interference**.)

double-slit interference

Consider two **in-phase** wave sources *of the same wavelength*, spaced a certain distance d apart. A location at a significant distance away from these two sources will experience either **constructive interference** or **destructive interference**, depending on the **path-length difference** ΔL between the distances each wave travels to meet at that location. In this case the **interference condition** is given by:

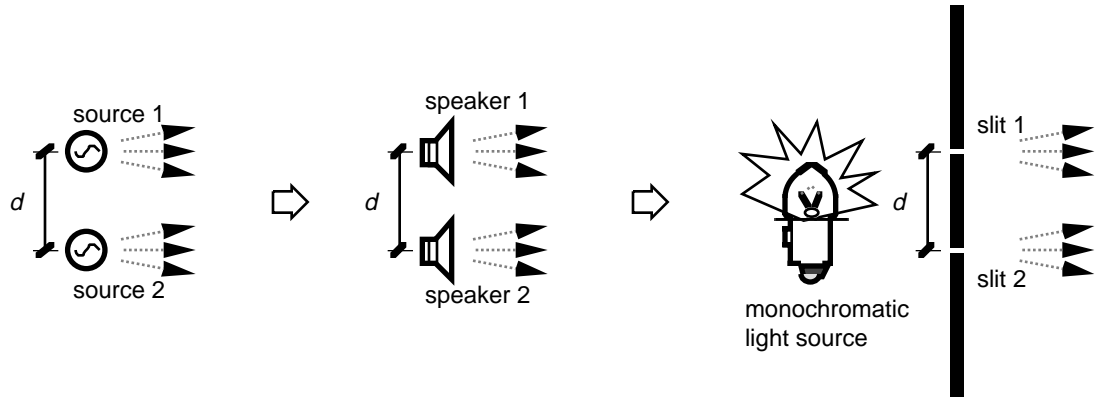
$$\Delta L = \begin{cases} (\#)\lambda & \text{constructive,} \\ (\# + \frac{1}{2})\lambda & \text{destructive.} \end{cases}$$

Thus merely comparing the path-lengths of each of the two waves is sufficient to determine whether they will interfere constructively or destructively.



The historical term "double-slit interference" follows from the original experiment performed by Thomas Young⁷, where a single wave incident on a barrier with two tiny slits cut into it, a distance d apart, will behave as two in-phase wave sources of the same wavelength. This experiment can be performed with water waves, light waves (of any given wavelength, and not just visible light wavelengths), or sound waves.

⁷ Thomas Young (1773-1829) was the first to conclusively prove that light behaves as a wave, as this experiment demonstrated that two light waves can undergo either constructive or destructive interference. The prevailing theory at the time was due to Isaac Newton (1672-1727), where light was considered to be comprised of "corpuscles" because it is observed to travel in straight lines.

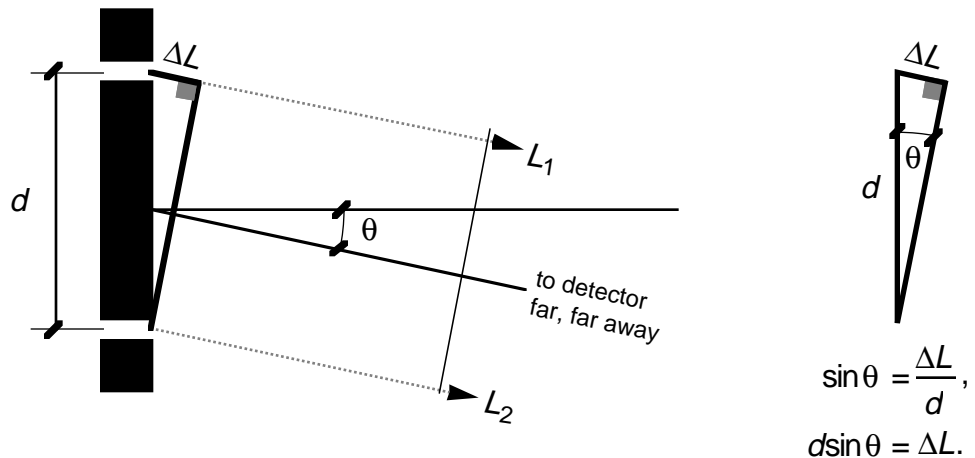


If the interference location is sufficiently far away from the two sources/slits, then the path-length difference ΔL can be expressed in terms of the angle θ between its location, and the $\theta = 0^\circ$ line that is perpendicular to the distance d between the slits.

$$\Delta L \approx d \sin \theta.$$

In this "far-field" approximation, the interference condition for a "far-field" point located along an arbitrary θ angle is given by:

$$d \sin \theta = \begin{cases} (\#)\lambda & \text{constructive,} \\ (\# + \frac{1}{2})\lambda & \text{destructive.} \end{cases}$$



(See **constructive interference**, **destructive interference**, **interference conditions**, **"in-phase,"** **path-length difference**.)

fundamental frequency

The lowest frequency that will excite a finite system to undergo **resonance**, and produce a **standing wave**.

For a rope (tied at both ends), or a closed-closed tube, or an open-open tube (or *any symmetric* system), their fundamental frequency is given by:

$$f_1 = \frac{v_{\text{wave}}}{2L},$$

where v_{wave} is the velocity of transverse waves along the rope, or the velocity of sound waves through air, and L is the distance between the ends of the system.

For a closed-open tube, or an open-closed tube (or *any asymmetric* system), their fundamental frequency is given by:

$$f_1 = \frac{v_{\text{wave}}}{4L},$$

where again, L is the distance between the ends of the system.

"**Harmonic frequencies**" are the sequence of frequencies higher than these fundamental frequencies that will also excite these systems to undergo resonance and produce standing waves. (*See "harmonic frequency," resonance, standing wave.*)

"harmonic frequency"

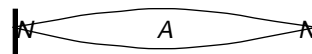
The sequence of frequencies above the **fundamental frequency** that will excite a finite system to undergo **resonance**, and produce a **standing wave**.

For a rope (tied at both ends), or a closed-closed tube, or an open-open tube (or *any symmetric* system), their harmonic frequencies are given by integer multiples of the fundamental frequency:

$$f_{\#} = (\#)f_1.$$

String instruments (such as pianos, harps, guitars, violins, *etc.*), brass instruments (such as trumpets, trombones, tubas, *etc.*), and "flutter-hole" wind instruments (such as piccolos, flutes, recorders, *etc.*) are all examples of symmetric resonant/standing wave systems that can play notes that are all integer multiples of a fundamental frequency. (Note that L can be changed as well on these instruments.)

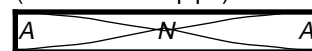
Fundamental
(closed-closed rope)



Fundamental (pressure)
(open-open pipe)

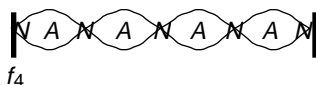
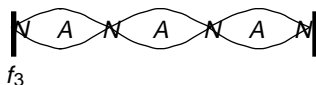
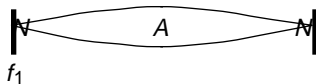
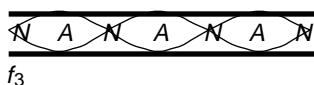
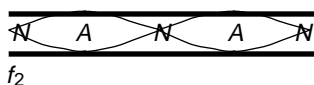
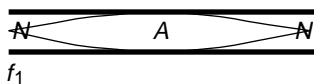
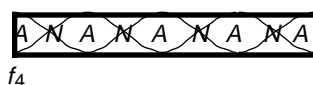
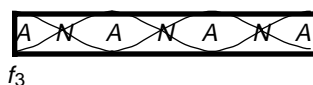
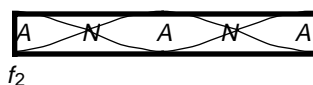
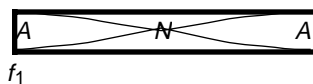
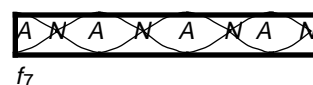
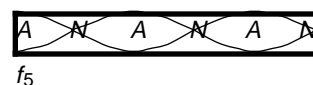
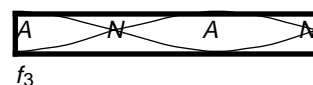


Fundamental (pressure)
(closed-closed pipe)



Fundamental (pressure)
(closed-open pipe)



Standing wave fluctuations
(closed-closed rope)Standing wave
pressure fluctuations
(open-open pipe)Standing wave
pressure fluctuations
(closed-closed pipe)Standing wave
pressure fluctuations
(closed-open pipe)

For a closed-open tube, or an open-closed tube (or *any asymmetric system*), their harmonic frequencies are given by *odd integer multiples* of the fundamental frequency:

$$f_{\text{odd}\#} = (\text{odd}\#)f_1,$$

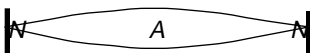
Reeded instruments (such as clarinets, saxophones, oboes, *etc.*) are all examples of asymmetric resonant/standing wave systems that can play notes that are *odd integer multiples* of a fundamental frequency. (Note that L can be changed as well. (See **fundamental frequency, resonance, standing wave.**)

node

A *node* is the location where two oppositely traveling waves of the same period and same wavelength *always* experience **destructive interference**. Observations of nodes depends on the specific type of **standing waves** involved:

- For a standing wave on a rope, every location where the rope is completely stationary ("looks pinched") is a node.
- For a standing sound wave (in air), every location where the gauge pressure of the air is always zero (no fluctuations) is said to be a pressure fluctuation node, as with an open end of a pipe.⁸

⁸ However, every location where the air molecules always remain at their equilibrium positions (no fluctuations) is said to be a position displacement node. These are *not* the same locations as pressure fluctuation nodes! This is because for a given sound wave, its pressure fluctuations and position displacements are shifted 90° from each other. For example, the closed end of a pipe is a pressure fluctuation **antinode**, as

Transverse fluctuations
(closed-closed rope)Pressure fluctuations
(open-open pipe)

(See **antinode**, **destructive interference**, **standing wave**.)

resonance

If a wave source is enclosed in a finite system, then the harmonic wave emitted from it will eventually come back to interfere with its own source. If the harmonic wave and its own source undergo **constructive interference**, then the system will *resonate*, such that the fluctuations put into the system will be continuously reinforced. Once a system undergoes resonance, it will produce a **standing wave**.

For the purposes of Block 12 in Physics 7C, we will consider two types of resonant systems: *symmetric* and *asymmetric*:

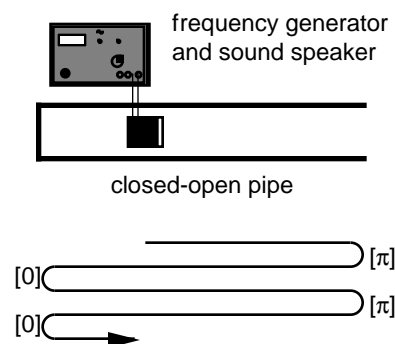
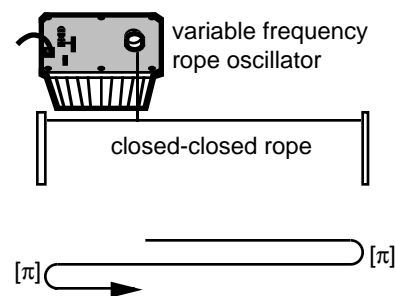
- For a *symmetric* system, the harmonic wave emitted from a source is reflected *two* times to come back in-phase with its own source, and each reflection has the same type of **reflection phase shift** (either both 0, or both π). For these systems there is a **node** at *both* ends, or an **antinode** at *both* ends. Its **fundamental frequency** is given by:

$$f_1 = \frac{v_{\text{wave}}}{2L},$$

where v_{wave} is the velocity of transverse waves along the rope, or the velocity of sound waves through air, and L is the distance between the (symmetric) ends of the system. Higher **harmonic frequencies** that excite resonance in this system are integer multiples of the fundamental frequency:

$$f_{\#} = (\#)f_1.$$

- For an *asymmetric* system, the harmonic wave emitted from a source must be reflected *four* times to come back in-phase to its own source, and the type of reflection at either end of the system has its own different reflection phase shift (one end reflects with a phase shift of π , the other end with a shift of 0, as for the gauge pressure fluctuations for sound waves in a closed-open pipe). For these systems there is a **node** at one end, and an **antinode** at the other end. Its **fundamental frequency** is given by:



air will be compressed and uncompressed there. However, the closed end of a pipe is a position displacement node, as you cannot displace the air molecules immediately next to the wall! Also consider an open end of a pipe, which is a position displacement antinode, as the air molecules there can easily be displaced from their equilibrium positions. However, the open end of a pipe is a pressure fluctuation node, as the end open to the atmosphere must *always* remain at atmospheric pressure!)

$$f_1 = \frac{v_{\text{wave}}}{4L},$$

where again, L is the distance between the asymmetric ends of the system. Higher harmonic frequencies that excite resonance in this system are odd multiples of the fundamental frequency:

$$f_{\text{odd}} = (\text{odd})f_1.$$

(See **antinode**, **fundamental frequency**, **"harmonic frequency," node**, **standing wave**.)

standing wave

Consider two harmonic waves of the *same period/wavelength* that travel in *opposite* directions. Certain locations will continuously undergo **constructive interference**, and certain locations will continuously undergo **destructive interference**. These special conditions are respectively the **antinodes** and **nodes** of the "standing" wave, because the antinode/node pattern produced by the superposition of these oppositely traveling, same period/wavelength waves appears to be completely stationary.

It would seem that **resonance** and standing waves are synonymous. For a finite closed system, resonant waves will make a standing wave, but not necessarily the other way around. A string that is vibrating (or caused to vibrate) at a resonant frequency will display a standing wave, with a node at each end of the string.

What about an infinite system, where we have two oppositely traveling harmonic waves of the same period/wavelength, each coming from their respective same-frequency sources at $-\infty$ and $+\infty$? In this case, we will also get a standing wave, with nodes and antinodes. *But this is not resonance, because in this unconstrained open system, each oppositely traveling wave is not returning back to its respective source to constructively interfere with itself.* Remember that a standing wave doesn't necessarily mean resonance occurs, but resonance makes a standing wave! In fact, for an infinite length string, with same-frequency sources at $-\infty$ and $+\infty$, *any* allowed frequency will make a standing wave. For a finite length string with just one source, *only* certain **harmonic frequencies** will make *resonant* standing waves. (See **antinode**, **constructive interference**, **destructive interference**, **"harmonic frequency," node**, **resonance**.)

thin film interference

Typically a light wave that encounters an interface between two media with different impedances will be partially reflected back from,

and partially transmitted into the new medium. If there is a transparent film that separates two different regions, there will be *two* (partially) reflected waves that will interfere with each other. The **interference condition** for whether these reflected waves will produce **constructive interference** or **destructive interference** will take on several different forms, depending on the relative impedances of the media involved:

- For a light wave that is incident on a high impedance film (of thickness t) that separates two lower impedance media, there will be two reflected waves that may interfere constructively or destructively according to the **path-length difference** ΔL :

$$\Delta L = 2t = \begin{cases} (\# + \frac{1}{2})\lambda_2 & \text{constructive,} \\ (\#)\lambda_2 & \text{destructive.} \end{cases}$$

Note that t is the thickness of medium 2, and λ_2 is the wavelength of light *inside of medium 2*.

This interference condition also applies for a light wave that is incident on a low impedance film (of thickness t) that separates two higher impedance media—the two reflected waves will interfere constructively or destructively according to the same interference condition given above.

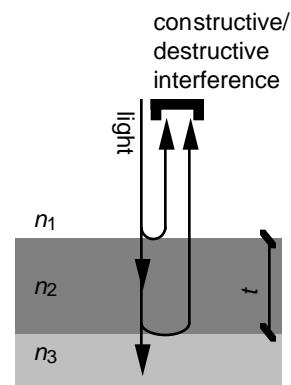
- For a light wave that is incident on two consecutive media of increasing impedance values, there will be two reflected waves that may interfere constructively or destructively as given by:

$$\Delta L = 2t = \begin{cases} (\#)\lambda_2 & \text{constructive,} \\ (\# + \frac{1}{2})\lambda_2 & \text{destructive.} \end{cases}$$

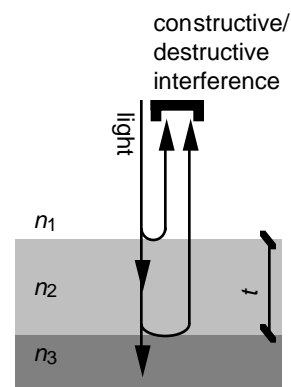
As before, t is the thickness of medium 2, and λ_2 is the wavelength of light *inside of medium 2*.

This interference condition also applies for a light wave that is incident on two consecutive media of *decreasing* impedance values—the two reflected waves will interfere constructively or destructively according to the same interference condition given above.

- Most generally, the universal interference condition for a light wave incident on a thin film system of *any* type (thickness and relative impedance values) is given in terms of their **total phase difference**:



$(n_2 > n_1, n_2 > n_3)$, or
 $(n_2 < n_1, n_2 < n_3)$



$(n_1 < n_2 < n_3)$, or
 $(n_1 > n_2 > n_3)$

$$\Delta\Psi = \left(-2\pi\Delta\left(\frac{L}{\lambda}\right) + \Delta\Psi_{\text{reflections}} \right) = \begin{cases} \pm(\text{even})\pi & \text{constructive,} \\ \pm(\text{odd})\pi & \text{destructive.} \end{cases}$$

Note that $\Delta\Psi_{\text{sources}} = 0$, as the two reflected light waves originated from the same source. The only parameters that contribute to the constructive or destructive interference behavior of the reflected waves come from the path-length difference and **reflection phase shift** terms.

Thin films are used as anti-reflective coatings on camera lenses and premium glasses and sunglasses. It turns out that the film atop these optics is the proper thickness in order for the light (of certain wavelengths) that is reflected off of the coating surfaces to destructively interfere. Thus if "no light" is reflected off of the front of the film, this means that all of the light is successfully transmitted *through* the thin film. The peculiarity of this system is that effectively no light is reflected back by the entire thin film system because of the destructive interference produced by the light reflected off of the top and bottom of the film itself! (See **constructive interference, destructive interference, interference conditions, reflection phase shift, total phase difference.**)

Block 13 Glossary

Forces and fields glossary

current (review)	force, magnetic (direct model)
direct model of forces	force, magnetic (field model)
electric field \vec{E}	gravitational field \vec{g}
electron (e^-)	magnetic field \vec{B}
field	neutron (n)
field model of forces	proton (p)
force	source object
force, electric (direct model)	test object
force, electric (field model)	vector superposition (review)
force, gravitational (direct model)	
force, gravitational (field model)	

current (review)

The transport of *positive* electric charge along a wire. Current is a vector; its *magnitude* measures the amount of charge that is transported per time past a certain point in the wire, and its *direction* is the direction of *positive* charge flow (or opposite the direction of *negative* charge flow):

$$I = \begin{cases} \text{magnitude} = \frac{\Delta Q}{\Delta t} \left[\frac{\text{Coul}}{\text{sec}} \right] \text{ or } [\text{Amps}] \\ \text{direction} = \text{positive charge flow} \end{cases}$$

In wires, positive metal ions are in their solid state and are not free to move, thus negative **electrons** move along the wire, and thus current is defined to be in the opposite direction of electron flow. (See **electron**.)

direct model of forces

Consider two interacting objects, A and B. The **source object** A exerts a force on the **test object** B. This is the **direct model of forces**, which is the quickest and simplest model of interaction between objects.



This is the interpretation of how forces worked in Physics 7B—we did not question *how* object A exerts a force on object B, we merely accepted the fact that it *does*.

It is important to note that whether the direct model of forces or the **field model of forces** is used; the result is the same—object A exerts a force on object B. (See **field model of forces, source object, test object.**)

electric field \vec{E}

(See **force (electric, field model).**)

electron (e^-)

A subatomic particle with a mass of 9.11×10^{-31} kg, and a charge of -1.602×10^{-19} Coulombs (which is *exactly* the negative amount of charge of a **proton**). The electron is much, much lighter in mass than either the proton or **neutron**. The electron is found in nature only outside the *nucleus* of an atom. The electron is denoted by " e^- ." (See **proton, neutron.**)

field

Recall that in Physics 7B we did not question *how* an object A is able to exert a force on another object B, we merely accepted the fact that it *does*. For the purposes of Block 13 in Physics 7C, we will model the *field* as the means by which an object A is able to exert a force on another object B (no matter how remotely located!).

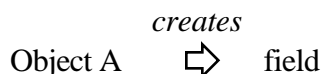
We will primarily focus on three types of fields in this Block: *gravitational fields, electric fields, and magnetic fields.*

- *Gravitational fields* are created by masses, and exert forces on other masses.
- *Electric fields* are created by charges, and exert forces on other charges.
- *Magnetic fields* are created by moving charges (such as currents), and exert forces on other moving charges.

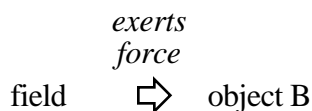
In the the **field model of forces**, the **source object** is the object that creates a field; the **test object** is what the field exerts a force upon. (The same object cannot be both a source object and a test object!) (See **direct model of forces, field model of forces, source object, test object.**)

field model of forces

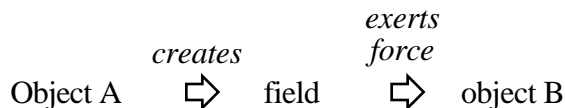
Consider two interacting objects A and B. If we consider object A as the **source object**, then object A will create a **field** in the space everywhere around it.



Now this field exists everywhere around object A, regardless of whether there is another object nearby or not. If we have a **test object** B located anywhere nearby, then the field (created by object A) will exert a force on object B.



Thus the complete field model of forces can be conceptually diagrammed as shown below, where the field is the "intermediary" that object A somehow uses to exert a force on another object B.



This is the model of forces used in Physics 7A—where the (source object) Earth created a gravitational field everywhere around it (especially at or near its surface); then this gravitational field exerted a force on other (test) objects.

It is important to note that whether the **direct model of forces** or the field model of forces is used; the result is the same—object A exerts a force on object B. (*See **direct model of forces, source object, test object.***)

force

A measure of the interaction between two objects that affects motion. In Physics 7B, force is expressed as a vector with a *magnitude* (expressed in units of Newtons) and *direction* (specified with an arrow). A force vector must be labeled in terms of a certain syntax:

$$\vec{F}_{(\text{type}) \text{ of } (\text{source object}) \text{ on } (\text{test object})}$$

We will primarily focus on three *types* of forces in this Block:

- **Gravitational forces** are interactions between masses M and m .
- **Electric forces** are interactions between stationary charges Q and q .
- **Magnetic forces** are interactions between moving charges (such as I currents, and q charges with a velocity \mathbf{v}).

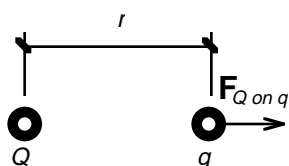
The **source object** is the object that does the exerting of a force; the **test object** is the object that the force is exerted upon. (The same object cannot be both a source object and a test object!)

We will also see how the means of which a source object exerts a force on a test object can be explained using two distinct, but closely related models—the **direct model of forces**, and the **field model of forces**. (See **direct model of forces, field model of forces, source object, test object**.)

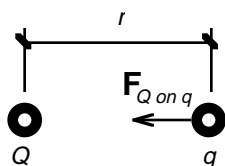
force, electric (direct model)

In the **direct model** of electric force, a **source charge** Q exerts a force on a **test charge** q . This direct model is the quickest way to calculate the magnitude and direction of electric forces.

$\text{Charge } Q \xrightarrow[\text{force}]{\text{exerts}} \text{charge } q$



Electric force for charges of same sign



Electric force for charges of opposite sign

The magnitude *and* direction of the electric force that the source charge Q exerts on a test charge q , separated by a center-to-center distance r , is given by:

$$\vec{\mathbf{F}}_{\text{electric of } Q \text{ on } q} = \begin{cases} \text{magnitude} = \left| k \frac{Qq}{(r_{Q \leftrightarrow q})^2} \right| \\ \text{direction} = \begin{pmatrix} \text{attractive if } \pm q, \mp Q \\ \text{repulsive if } \pm q, \pm Q \end{pmatrix} \end{cases}$$

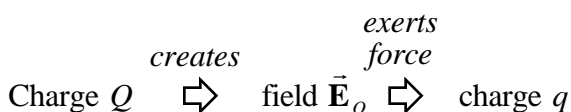
The constant $k = 8.99 \times 10^9 \text{ N} \cdot \text{m}^2/\text{Coul}^2$ converts everything to the proper units of Newtons. Note that this force can be repulsive as well as attractive, depending on the signs of the two charges $\pm Q$ and $\pm q$. Like sign charges repel, but opposite sign charges attract, as shown at left.

Electric charges are either positive or negative, and are measured in Coulombs. The **electron** has a charge of

-1.602×10^{-19} Coulombs, and whether an atom, molecule, or macroscopic object has a net positive or negative charge depends on whether it has a surplus or deficit of electrons (compared to the amount of **protons** it has, each of which has a charge of $+1.602 \times 10^{-19}$ Coulombs). This amount of charge is said to be *fundamental*, as all observed particles or objects have some integer multiple of $\pm 1.602 \times 10^{-19}$ Coulombs. (See **direct model of forces, electron, proton, source object, test object.**)

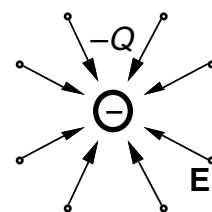
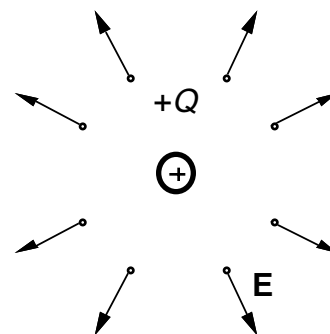
force, electric (field model)

In the **field model** of **electric force**, the **source charge Q** creates an **electric field vector \vec{E}_Q** at each and every point in space around itself. Then this electric field exerts a force on a **test charge q** .



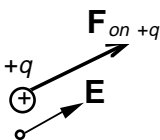
The *magnitude* (measured in Newtons/Coulomb) and *direction* of the electric field vector \vec{E}_Q created by the source charge Q are given by:

$$\vec{E}_{\text{of source charge } Q} = \begin{cases} \text{magnitude} = \left| k \frac{Q}{r^2} \right| \\ \text{direction} = \begin{pmatrix} \text{in towards } -Q \\ \text{out away from } +Q \end{pmatrix} \end{cases}$$



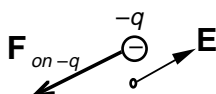
The distance r is measured from the center of the source object, to the specific location in space being considered. The constant $k = 8.99 \times 10^9 \text{ N} \cdot \text{m}^2/\text{Coul}^2$ converts everything to the proper units of Newtons/Coul. Note that the magnitude of the \vec{E} field is an absolute value—it is just a length of a vector; the direction of the \vec{E} field vector (in towards or out away from Q) is determined by inspecting the \pm sign of the source charge Q , as shown for the two cases at right.

The *magnitude* and *direction* of the electric force exerted by the \vec{E} field on a test charge q is then given by:



$$\vec{F}_{\text{of } \vec{E} \text{ field on test charge } q} = \begin{cases} \text{magnitude} = qE; \\ \text{direction} = \begin{cases} \text{along } \vec{E} \text{ for } +q \\ \text{opposite } \vec{E} \text{ for } -q \end{cases} \end{cases}$$

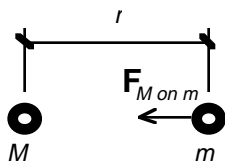
So the direction of this electric force on the test charge q depends on the \pm sign of the test charge q —the force on it will be the same direction as the electric field vector if the test charge q is positive, or will be in the opposite direction if the test charge q is negative.



Note that there are two distinct steps in the field model of electric forces: (i) the source charge Q creates an electric field \vec{E}_Q ; then (ii) the electric field \vec{E}_Q exerts a force on a test charge q . (See **field model of forces, source object, test object.**)

force, gravitational (direct model)

In the **direct model** of **gravitational force**, the **source mass** M exerts a force on a **test mass** m . The direct model is the quickest way to calculate the magnitude and direction of gravitational forces.



Gravitational force
for two masses

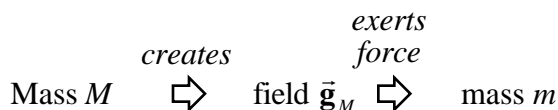
The magnitude *and* direction of the electric force that the source mass M exerts on a test mass m , separated by a center-to-center distance r , is given by:

$$\vec{F}_{\text{gravitational of } M \text{ on } m} = \begin{cases} \text{magnitude} = \left| G \frac{Mm}{r^2} \right| \\ \text{direction} = \text{in towards } M \end{cases}$$

The constant $G = 6.67 \times 10^{-11} \text{ N} \cdot \text{m}^2 / \text{kg}^2$ (the "universal gravitational constant," which should be called "big G " in order to distinguish it from the "little g " **gravitational field** \vec{g}) converts everything to the proper units of Newtons. Gravitational forces are *always* attractive. (See **direct model of forces, source object, test object.**)

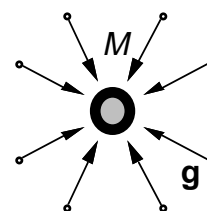
force, gravitational (field model)

In the **field model** of **gravitational force**, a **source mass M** creates a **gravitational field vector \vec{g}_M** at every point in space around itself. Then this gravitational field exerts a force on a **test mass m** .



The *magnitude* (measured in Newtons/kilogram) and *direction* of the gravitational \vec{g}_M field vectors created by the source mass M are given by:

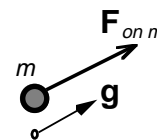
$$\vec{g}_{\text{of source mass } M} = \left\{ \begin{array}{l} \text{magnitude} = \left| G \frac{M}{r^2} \right| \\ \text{direction} = \text{in towards } M \end{array} \right.$$



The distance r is measured from the center of the source object, to the specific location in space being considered. The constant $G = 6.67 \times 10^{-11} \text{ N} \cdot \text{m}^2 / \text{kg}^2$ (the "universal gravitational constant") converts everything to the proper units of Newtons/kilogram. Gravitational field vectors *always* point in towards their source mass M .

The *magnitude* and *direction* of the gravitational force exerted by the \vec{g} field on a test mass m is then given by:

$$\vec{F}_{\text{of } \vec{g} \text{ field on test mass } m} = \left\{ \begin{array}{l} \text{magnitude} = mg \\ \text{direction} = \text{along } \vec{g} \end{array} \right.$$

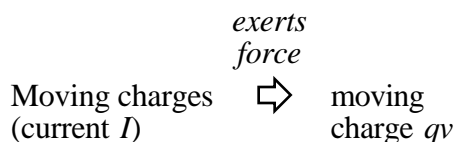


The direction of this gravitational force on the test mass m always lies along the same direction as a gravitational field vector.

Note that there are two distinct steps in the field model of gravity: (i) the source mass M creates a gravitational field \vec{g}_M ; then (ii) the gravitational field \vec{g}_M exerts a force on a test mass m . (*See field model of forces, source object, test object.*)

force, magnetic (direct model)

In the **direct model** of **magnetic force**, a **source current I** exerts a force on a **test charge q** that moves with a velocity v .



For the purposes of Block 13 in Physics 7C, we will only consider the **field model** of magnetic forces. (See **current, direct model of forces, force (magnetic, field model), source object, test object.**)

force, magnetic (field model)

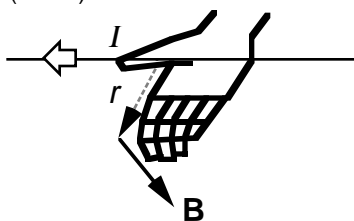
In the **field model** of magnetic force, the **source current** I creates a **magnetic field vector** \vec{B}_I at every point in space around it. This magnetic field may exert a force on a **test charge** q , if that charge has a non-zero velocity \vec{v} .

$$\begin{array}{ccccc} & & \text{creates} & & \text{exerts} \\ & & & & \text{force} \\ \text{Moving charges} & \Rightarrow & \text{field } \vec{B}_I & \Rightarrow & \text{on other moving} \\ (I \text{ currents}) & & & & \text{charges } qv \end{array}$$

The *magnitude* (measured in units of Teslas) and *direction* of a \vec{B}_I field vector a distance r away from a wire carrying a current I are given by:

$$\vec{B}_{\text{of source current } I} = \left\{ \begin{array}{l} \text{magnitude} = \left| \frac{\mu_0 I}{2\pi r} \right| \\ \text{direction} = \text{RHR1} \end{array} \right.$$

Right hand rule for \vec{B} field vectors (RHR1)



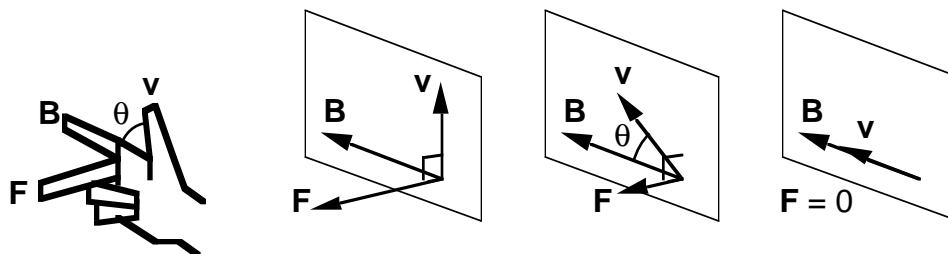
The distance r is measured from the center of the source object (current-carrying wire), to the specific location in space being considered. The constant $\mu_0 = 1.26 \times 10^{-6}$ Tesla \cdot m / Amps converts everything to the proper units of Teslas. The direction of the \vec{B}_I field vector created by the current is given by the first of two "right-hand rules," as shown at left. Align the thumb of your right hand along the direction of current I , as in a wire, then your fingers will indicate the direction of the \vec{B}_I field vectors around it. The angle between r and \vec{B}_I is *always* 90° . (Remember that electric current I is defined to be the direction of *positive* charge flow.)

The *magnitude* and *direction* of the magnetic force exerted by the \vec{B} field on a moving test charge q with a velocity \vec{v} is then given by:

$$\vec{F}_{\text{of } \vec{B} \text{ field on moving test charge } qv} = \left\{ \begin{array}{l} \text{magnitude} = |qvB \sin \theta_{\vec{v}}^{\vec{B}}| \\ \text{direction} = \left(\begin{array}{l} \text{along RHR2 for } +q \\ \text{opposite RHR2 for } -q \end{array} \right) \end{array} \right.$$

The direction of the magnetic force on the moving test charge q is given by the second "right-hand rule," as shown below. The first

three fingers of your right hand should nominally all be at right angles to each other. Align your thumb along the velocity vector \mathbf{v} of the moving test charge q , with your index (pointer) finger along the magnetic field vector \mathbf{B} , and your middle finger should point along the direction of the magnetic force on the moving test charge q .



Right-hand rule for magnetic forces on positive moving electric charges (RHR2)

The ultimate direction of this magnetic force depends on the \pm sign of the test charge q —the force on it will be in the direction given by the RHR2 if the q charge is positive, or will be in the opposite direction of RHR2 if the q charge is negative. Note that θ is the angle between \mathbf{v} and \mathbf{B} . The magnetic force is a maximum when $\theta = 90^\circ$, less than this if $\theta < 90^\circ$, and the magnetic force is completely zero if $\theta = 0^\circ$ (or 180°). *Also important is the fact that the test charge must be moving for it to experience a magnetic force on it—if the velocity of the charge is $\mathbf{v} = 0$, then there is no magnetic force on the electric charge!*

Note that there are two distinct steps in the field model of magnetic forces: (i) the source current I creates a magnetic field \mathbf{B} ; then (ii) the magnetic field \mathbf{B} exerts a force on a moving test charge q . (See **current, field model of forces, source object, test object.**)

gravitational field \vec{g}

(See **force (gravitational, field model).**)

magnetic field \vec{B}

(See **force (magnetic, field model).**)

neutron (n)

A subatomic particle with a mass of 1.6750×10^{-27} kg, and zero charge. (The neutron is slightly more massive than the **proton**.) Together with the proton, the neutron is found in nature only within the *nucleus* of an atom. The neutron is denoted by "n." (See **electron, proton.**)

proton (p)

A subatomic particle with a mass of 1.6726×10^{-27} kg, and a charge of $+1.602 \times 10^{-19}$ Coulombs (which is *exactly* the positive amount of

charge of an **electron**). The proton is slightly less massive than the **neutron**. Together with the neutron, the proton is found in nature only within the *nucleus* of an atom. Chemically, a proton should already be familiar to you as the H^+ ion. The proton is denoted by just "p." (See **electron, neutron**.)

source object

In the **direct model of forces**, the **source object** exerts a force on a **test object**. In the **field model of forces**, the source object creates a field everywhere all around it. In either model, it is really important to be able to distinguish the difference between a source object and a test object. (See **direct model of forces, field model of forces**.)

test object

In the **direct model of forces**, the **source object** exerts a force on a **test object**. In the **field model of forces**, a field exerts a force on a test object. In either model, it is really important to be able to distinguish the difference between a source object and a test object. (See **direct model of forces, field model of forces**.)

vector superposition (review)

In Blocks 7-9 of Physics 7B, **force** vectors were the only type of vectors that are physically meaningful to add (superpose together). We added together all the \vec{F} forces that acted on a given object, in order to find out the *net force* $\sum \vec{F}$ that acted on that object. We will be adding force vectors in Block 13 of Physics 7C, and later in this Block we will be adding **field** vectors as well.

There are two ways of adding up vectors. For the purposes of Block 13 in Physics 7C, we will be using *only tail-to-head addition*. Simpler yet, we will be considering only addition of one-dimensional vectors.

In *tail-to-head addition*, multiple vectors are added together by lining them up into a chain of tail-to-head vectors. Remember that it's okay to move vectors around, as long as they keep the *same* magnitude and direction. The *resultant* vector is drawn last, with its tail at the tail of the tail-to-head chain; the head is at the head of the tail-to-head chain. It doesn't matter in what order vectors are added tail-to-head, the net force vector will be the same. (See **field, force**.)

Fields and potentials glossary**gradient relation****"Lennard-Jones potential"****potential energy****potential energy, electric**

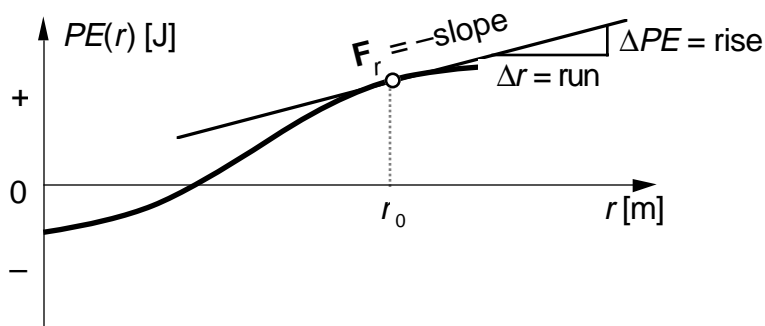
$$PE_{elec}$$

potential energy,**gravitational PE_{grav}** **potential energy, inter-atomic****potential energy, magnetic****work (review)****gradient relation**

A differential relation between the force exerted on a test object, and its resulting change in the corresponding **potential energy** as the test object is moved from an initial location $r_{initial}$ to a final location r_{final} :

$$F_{along\ r} = -\frac{\Delta PE}{\Delta r} = -\frac{(PE_{final} - PE_{initial})}{(r_{final} - r_{initial})},$$

where the negative sign indicates that the force on a test object points in the direction that will *reduce* its potential energy. This relation merely follows from the definition of **work** used in Physics 7A. (Note that a change in position may also be Δx or Δy , etc.)



This relation has an interesting interpretation for a graph of the potential energy of a test object plotted versus the position of this object. The slope of this graph, at a certain position, gives the magnitude of the force exerted on a test object located there. The direction of the force on the test object can be found by inspection—literally think of a ball resting on the slope. Whether this conceptual ball rolls downwards to the left or to the right indicates whether the force is directed to the left or to the right. In the example above, the force would be directed to the left. (See **potential energy, work**.)

"Lennard-Jones potential"(See **potential energy (inter-atomic)**.)

potential energy

Since we concentrate on three types of interactions between source and test objects (gravitational, electric, and magnetic) in Block 13 of Physics 7C, here we will consider three fundamental types of potential energy that can be stored by a test object, at some location relative to a source object: **gravitational potential energy** PE_{grav} , **electric potential energy** PE_{elec} , and **magnetic potential energy** PE_{mag} (although strictly speaking, $\Delta PE_{mag} = 0$ for all processes, and thus becomes irrelevant). All forms of potential energies obey a **gradient relation** between their slopes and the forces exerted on the test object by the source object. (*See gradient relation, potential energy (electric), potential energy (gravitational), potential energy (magnetic).*)

potential energy, electric PE_{elec}

Electric potential energy PE_{elec} is stored in the position of a test charge q , relative to a source charge Q . For a test charge q moved from an initial location $r_{initial}$ to a final location r_{final} , as measured from a source charge Q , its change in electrical potential energy is given by:

$$\Delta PE_{elec} = kQq\Delta\left(\frac{1}{r}\right) = kQq\left(\frac{1}{r_{final}} - \frac{1}{r_{initial}}\right),$$

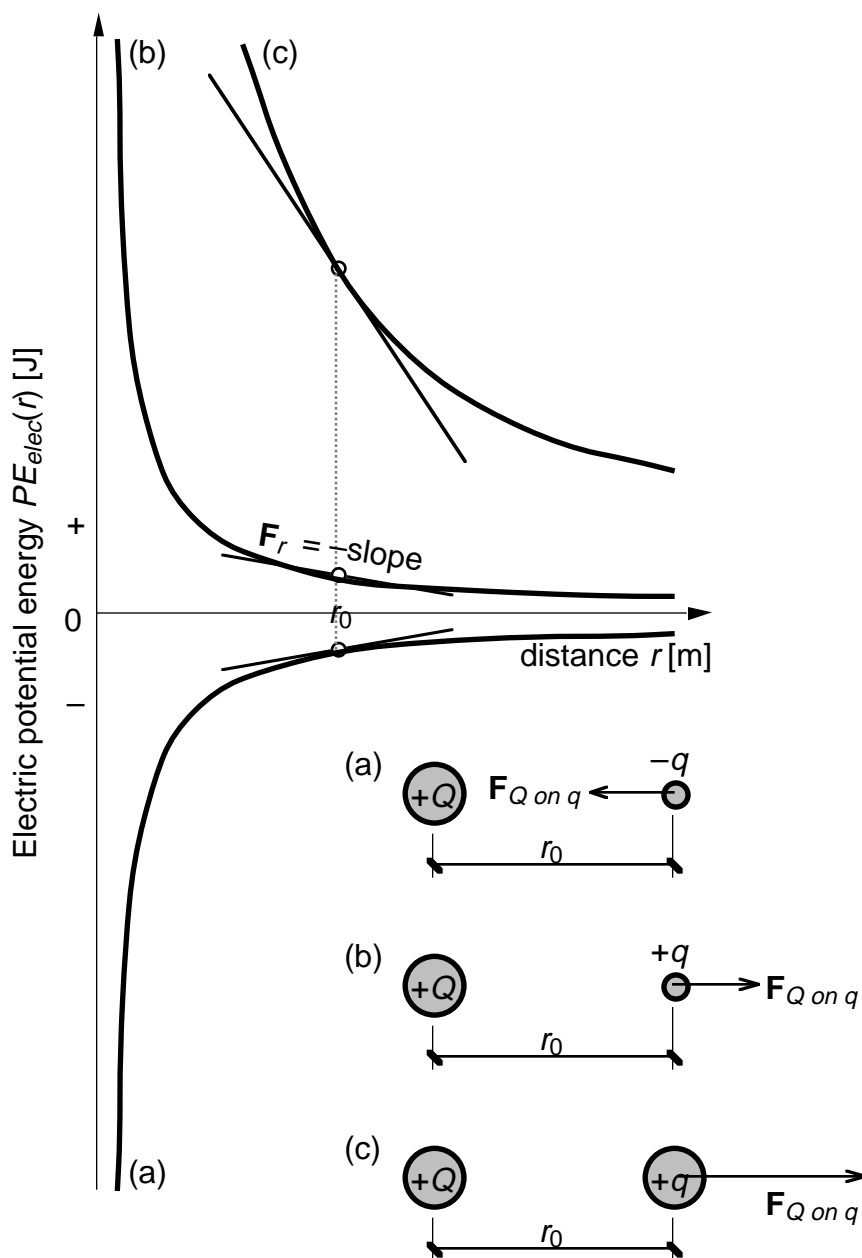
where the constant $k = 8.99 \times 10^9 \text{ N} \cdot \text{m}^2/\text{Coul}^2$ and r is the center-to-center distance between the source charge Q and the test charge q . All \pm signs for Q and q must be explicitly included in the above equation! Electrical potential energy *increases* when like charges are brought closer to each other; or when opposite charges are moved farther away from each other. Electrical potential energy *decreases* when like charges are moved farther away from each other; or when opposite charges are brought closer to each other.

Shown on the next page is a plot of the electric potential energy of a test charge q , as a function of the center-to-center distance r between Q and q . Note that the usual convention is to set $PE_{elec} = 0$ for $r = \infty$, and to hold the charge on the left (Q) stationary, while varying the position of the charge on the right (q). Example (a) shows the PE_{elec} curve for a $+Q$ charge and a $-q$ charge. At a distance r_0 between their centers, the *magnitude* of the electric force exerted on the $-q$ charge is the tangent slope of the PE_{elec} curve at r_0 (as determined by the **gradient relation**); the *direction* of the electric force is towards the left, which would *decrease* PE_{elec} .

Example (b) shows the PE_{elec} curve for a $+Q$ charge and a $+q$ charge, where the test charge has the same amount of charge as before, but positive instead of negative. At a distance r_0 between

their centers, the *magnitude* of the electric force exerted on the $+q$ charge is the tangent slope of the PE_{elec} curve at r_0 (which is the same as in example (a)), but in this case the *direction* of the electric force is towards the right, which would *decrease* PE_{elec} .

Example (c) shows the PE_{elec} curve for a $+Q$ charge and a $+q$ charge, where the test charge now has a greater amount of positive charge. At a distance r_0 between their centers, the *magnitude* of the electric force exerted on the $+q$ charge is the steeper tangent slope of the PE_{elec} curve at r_0 (which is greater than in examples (a) and (b)); and in this case the *direction* of the electric force is towards the right, which would *decrease* PE_{elec} . (See **gradient relation, potential energy.**)



potential energy, inter-atomic

In Physics 7A, the "Lennard-Jones potential" was used to describe how atoms interact with each other, whether they be bound together in a solid, brushing past each other in a liquid, or freely flying past each other in a gaseous phase.

The "Lennard-Jones potential" is the resulting potential energy of two interacting atoms, which behave in a much more complex manner than two static balls of charge, due to two competing effects. First there is a repulsive force between the positively charged nucleus of either atom, although this effect is only important when the atoms get very, very close to each other. Secondly, when the atoms are just a little farther away from each other, then there is an attractive force between them, due to the quantum-mechanical interaction of bonding (and non-bonding) atomic orbitals.

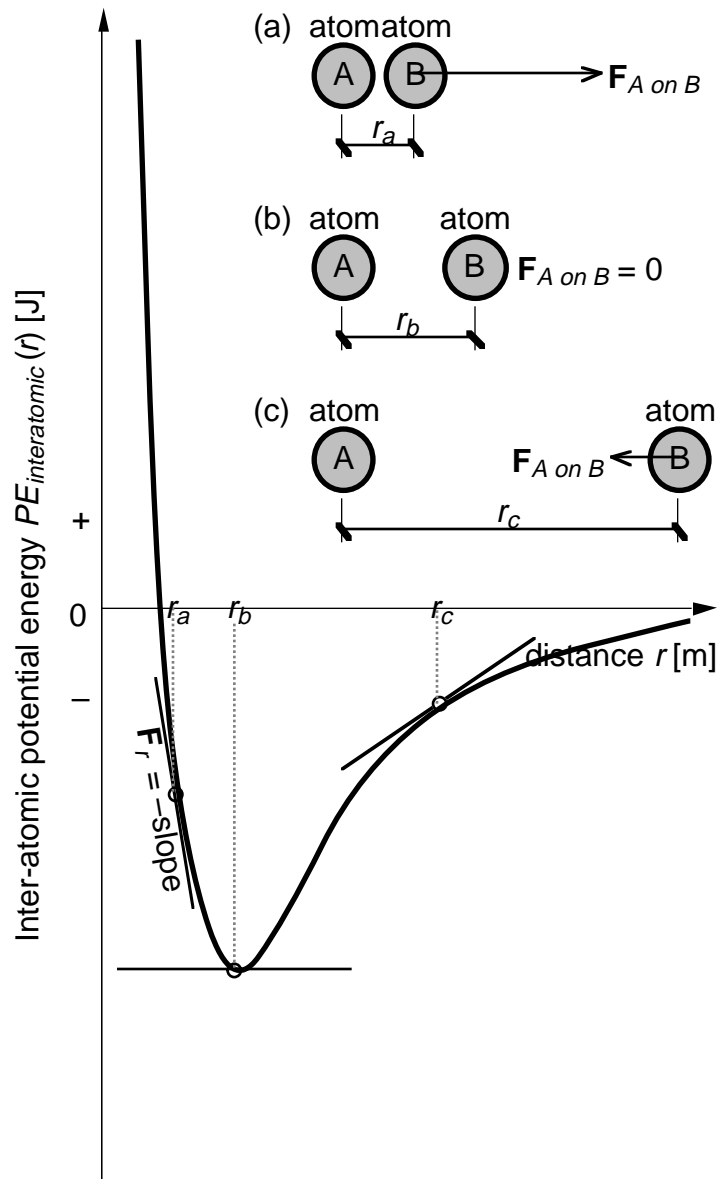
The superposition of this close-together repulsion and far-away attraction of two atoms for all r distances between them results in the familiar inter-atomic potential energy graph shown on the next page, as a function of the center-to-center distance r between the two atoms. Any and all atom-atom interactions will have this general shape; to make this graph specific for a certain type of inter-atomic interaction only requires specifying the depth of the curve, and the r_0 distance of the equilibrium position of the curve. Note that the usual convention is to set $PE_{inter-atomic} = 0$ for $r = \infty$, and to hold the atom A on the left stationary, while varying the position of the atom B on the right.

Remember that the **gradient relation** between forces and PE slopes applies here as well. The *magnitudes* of the forces exerted between these interacting atoms can be calculated from the slopes of the $PE_{inter-atomic}$ graph; the *directions* of the forces involved must point towards decreasing potential energy.

At the very close inter-atomic distance r_a from each other, the *magnitude* of the force of atom A on atom B is the tangent slope of the $PE_{inter-atomic}$; the *direction* of this force is towards the right, which would *decrease* $PE_{inter-atomic}$.

At the equilibrium inter-atomic distance r_b from each other, the tangent slope of the $PE_{inter-atomic} = 0$, thus the *magnitude* of the force of atom A on atom B is zero.

At the very close inter-atomic distance r_c from each other, the *magnitude* of the force of atom A on atom B is the tangent slope of the $PE_{inter-atomic}$ (which is less than at r_a); the *direction* of this force is towards the left, which would *decrease* $PE_{inter-atomic}$. (See **gradient relation, potential energy.**)



potential energy, gravitational PE_{grav}

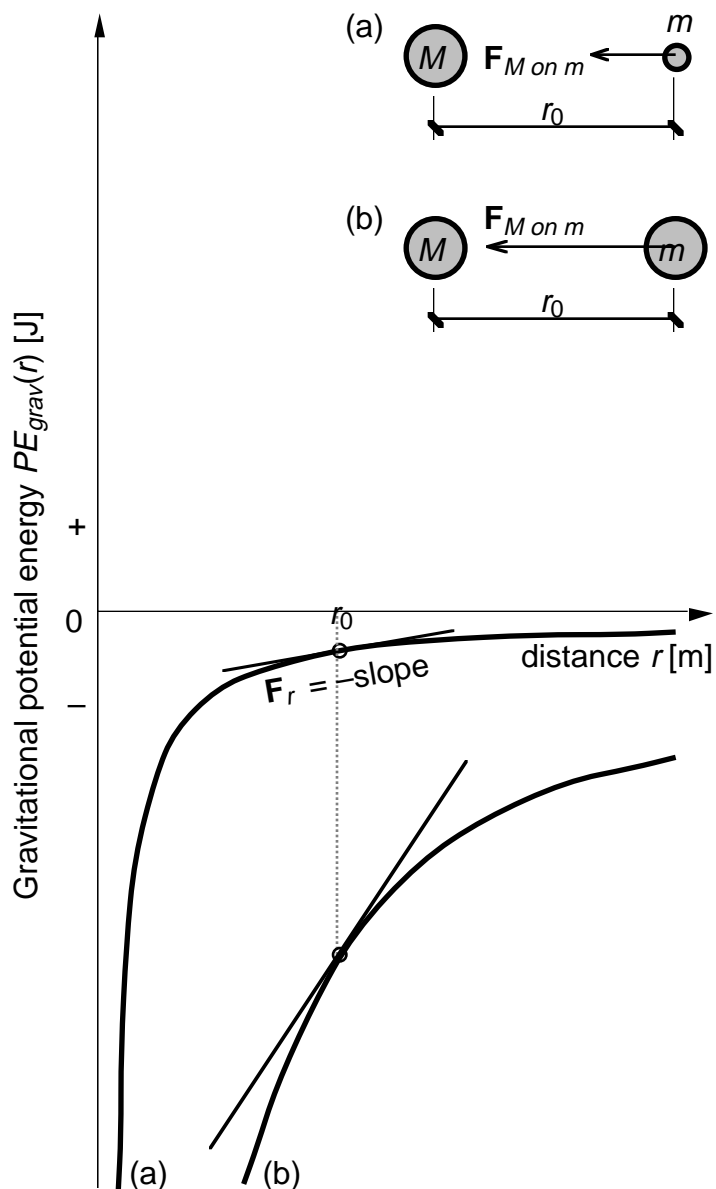
Gravitational potential energy PE_{grav} is stored in the position of a test mass m , relative to a source mass M . For a test mass m moved from an initial location $r_{initial}$ to a final location r_{final} , as measured from a source mass M , its change in gravitational potential energy is given by:

$$\Delta PE_{grav} = -GMm\Delta\left(\frac{1}{r}\right) = -GMm\left(\frac{1}{r_{final}} - \frac{1}{r_{initial}}\right),$$

where the constant $G = 6.67 \times 10^{-11} \text{ N} \cdot \text{m}^2 / \text{kg}^2$, and r is the center-to-center distance between the source mass M and the test mass m .

Gravitational potential energy *increases* when masses are moved farther away from each other. Gravitational potential energy *decreases* when masses are brought closer to each other.

Shown below is a plot of the gravitational potential energy of a test mass m , as a function of the center-to-center distance r between M and m . Note that the usual convention is to set $PE_{grav} = 0$ for $r = \infty$, and to hold the mass on the left (M) stationary, while varying the position of the mass on the right (m). Example (a) shows the PE_{grav} curve for a mass M and a mass m . At a distance r_0 between their centers, the *magnitude* of the gravitational force exerted on m is the tangent slope of the PE_{grav} curve at r_0 (as determined by the **gradient relation**); the *direction* of the gravitational force is towards the left, which would *decrease* PE_{grav} .



Example (b) shows the PE_{grav} curve for a mass M and a mass m , where the test mass now has a greater amount mass. At a distance r_0 between their centers, the *magnitude* of the gravitational force exerted on the mass m is the steeper tangent slope of the PE_{grav} curve at r_0 (which is greater than in example (a)); and in this case the *direction* of the gravitational force is still towards the left, which would *decrease* PE_{grav} . (See **gradient relation, potential energy.**)

potential energy, magnetic

Although we could define a magnetic potential energy PE_{mag} , for all possible processes we will encounter in Block 13 in Physics 7C, $\Delta PE_{mag} = 0$ (can you give an example why this would be so?). (See **potential energy, work.**)

work (review)

The transfer of energy into or out of a system, due to a force exerted on a test object, over a certain change in position Δr (or Δx , Δy , *etc.*). The force must point along the change in position (if not, then only the component of force that points along the change in position is considered):

$$Work = F_{\text{along } x} \Delta x.$$

Here we will consider forces (gravitational and electric) that are exerted in directions that point towards (or away) from the center of source objects. Thus the relevant change in position will be in the radial direction (Δr), as measured directly from the center of the source object, and the energy system that would have work done on it would be potential energy:

$$Work = F_{\text{along } r} \Delta r,$$

$$-\Delta PE = F_{\text{along } r} \Delta r.$$

The negative sign here indicates that a gravitational, electric, or an inter-atomic force on a test object will point in the direction that would *reduce* the potential energy of the test object.

The peculiar thing is that magnetic forces can never point along the direction v that the test charge q is moving—convince yourself of this using RHR2. Because of this, there is no parallel component of magnetic force along the change in position of the test charge, and there is "no magnetic work" exerted on the moving test charge.

So we consider gravitational, electric, and inter-atomic work because they are *radial* forces; there is no such thing as magnetic work because it is *not* a radial force. (See **gradient relation, potential energy.**)

Block 14 Glossary

Nuclear fusion/fission glossary

"assembly/disassembly energy"	neutron (n)
atomic mass unit (amu)	neutron number N
"atomic number" Z	nuclear radius R
binding energy	nuclear reaction
"binding energy curve"	nuclear strong interaction
chemical reaction	nuclear strong bond
electron (e^-)	nuclear strong potential energy
electron-volt (eV)	nucleon
femtometer (fm)	nucleon number A
fission	nucleus
four fundamental interactions of nature	partons
fusion	potential energy
kinetic energy KE	potential energy, inter-nuclear
mass decrement	proton (p)
mass defect	proton number Z
mass-energy equivalence	Q -value
"mass number" A	unified atomic mass unit (u)
megaelectron-volt (MeV)	

"assembly/disassembly energy"

(See **binding energy**.)

atomic mass unit (amu)

An obsolete unit of mass, not to be confused with its replacement, the **unified atomic mass unit** (u). A word of caution—current physics and chemistry textbooks may not adequately distinguish the difference between amu and u units. (See **unified atomic mass unit**.)

"atomic number" Z

(See **proton number**.)

binding energy

Recall that the **mass defect** is the difference between the mass of a **nucleus**, and the mass of all of its **neutrons** and **protons** added together. If this mass defect is multiplied by the speed of light squared, due to **mass-energy equivalence** one obtains the binding energy of the nucleus—that is, the amount of energy released when this nucleus is formed from constituent raw neutrons

and protons (or conversely, the energy required to be put in to break up the nucleus into its separate neutrons and protons):

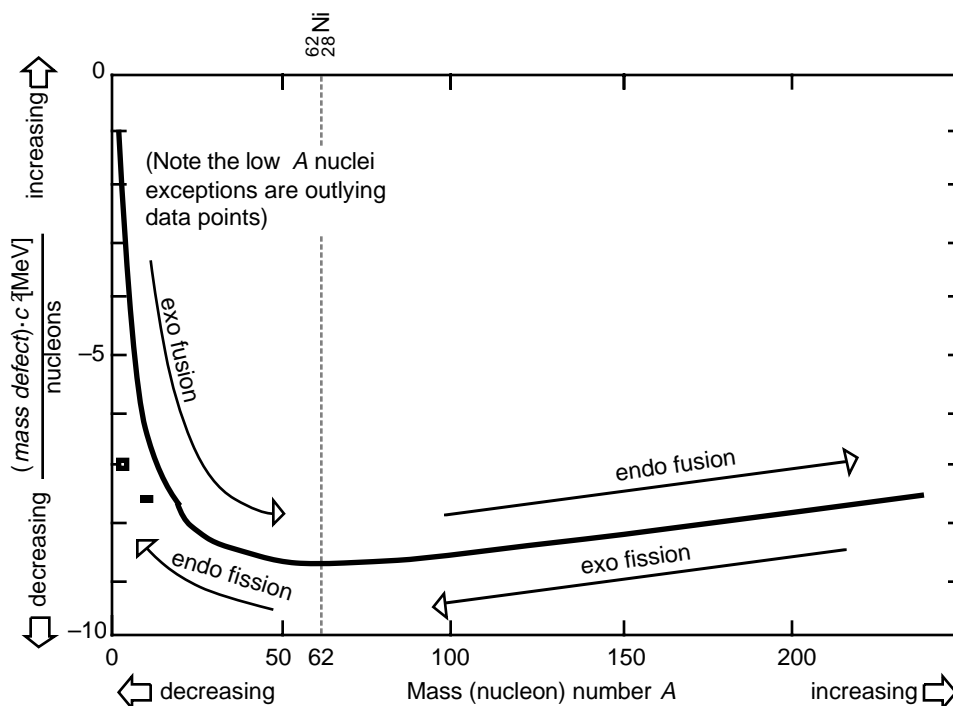
$$\begin{aligned} \text{"binding energy"} &= (\text{mass defect})c^2, \\ &= (m_{\text{nucleus}} - m_{\text{protons}} - m_{\text{neutrons}})c^2, \end{aligned}$$

where $c = 3 \times 10^8$ m/s is the magnitude of the velocity of light in a vacuum. Note that with this definition, the binding energy of all nuclei is a *negative* quantity, and that heavier nuclei have binding energies with bigger negative values. Sometimes binding energy is referred to as *assembly energy* (or *disassembly energy*) because it is the "gain" (or "cost") of putting a nucleus together from (or breaking it apart into) its constituent protons and neutrons. (See **binding energy curve**, **mass defect**, **mass-energy equivalence**, **neutron**, **nucleus**, **proton**.)

"binding energy curve"

Recall that heavy nuclei have the greatest (negative) binding energies. However, if we plot **binding energy** per nucleon number A on the vertical axis, versus **nucleon number** A on the horizontal axis, we obtain a binding energy curve, shown below (actually, it should properly be called the "binding energy per nucleon versus nucleon curve").

Note that the minimum of this binding energy curve is at $A = 62$, for the ${}_{28}^{62}\text{Ni}$ nucleus. This means that the ${}_{28}^{62}\text{Ni}$ nucleus releases the most energy (*per nucleon*) when it is formed from raw protons and



neutrons; or that the ${}_{28}^{62}\text{Ni}$ nucleus requires the most energy (*per nucleon*) in order to break it up into its constituent neutrons and protons.

By comparing the binding energies of the reactants and products of a nuclear reaction, and seeing if it increases or decreases, then the reaction can be determined to be endothermic or exothermic, respectively. (See **binding energy, nucleon number, Q-value.**)

chemical reaction

A process that rearranges the molecular configuration of atoms, by changing the configuration of the electron orbitals surrounding each atom. The atoms themselves do not change; merely their arrangement relative to each other does, in order to break apart and/or make molecules. (See **nuclear reaction.**)

electron (e^-)

A subatomic particle with a mass of 9.11×10^{-31} kg (or 0.0005486 u), and a charge of -1.602×10^{-19} Coulombs (which is *exactly* the negative amount of charge of a **proton**). The electron is much, much lighter in mass than either the proton or **neutron**. The electron is found in nature only outside the **nucleus** of an atom. The configuration of atomic electron orbitals determines the chemical properties of an atom. The electron is denoted by ${}^0_{-1}\text{e}$, or sometimes just " e^- ." The **antimatter** twin of an electron is the positively charged **positron**. (See **antimatter, electron, neutron, nucleus, positron, unified atomic mass unit.**)

electron-volt (eV)

A non-standard unit of energy, where $1 \text{ eV} = 1.602 \times 10^{-19}$ J. Historically, electron-volts are used because they are convenient in describing the energies involved in typical laboratory experiments (as in accelerating an electron across a 1 Volt potential difference), and atomic and molecular orbital transitions, which are given in the range of hundredths to hundreds of electron-volts of energy. Nuclear reactions are typically given in the range of millions of electron-volts of energy. (See **megaelectron-volt.**)

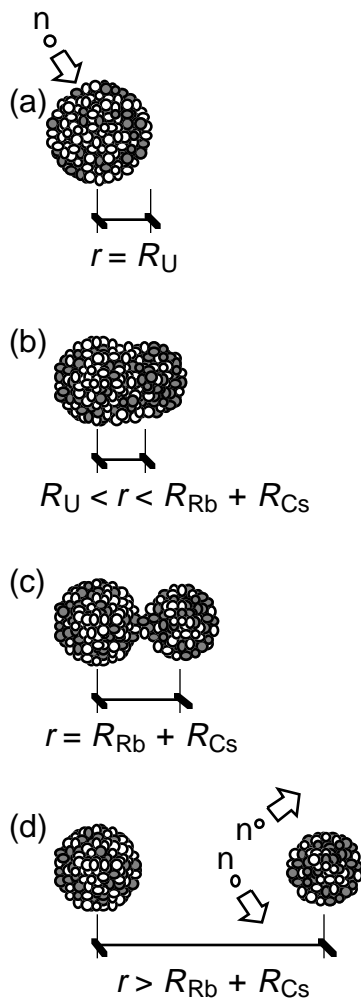
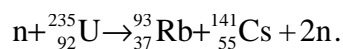
femtometer (fm)

A non-standard unit of length, where $1 \text{ fm} = 1 \times 10^{-15}$ m. Historically, femtometers are used because they are convenient in describing the distances involved in nuclear reactions, where the radii of nuclei (and the center-to-center distances between nuclei) are typically given in femtometers. (See **nuclear radius.**)

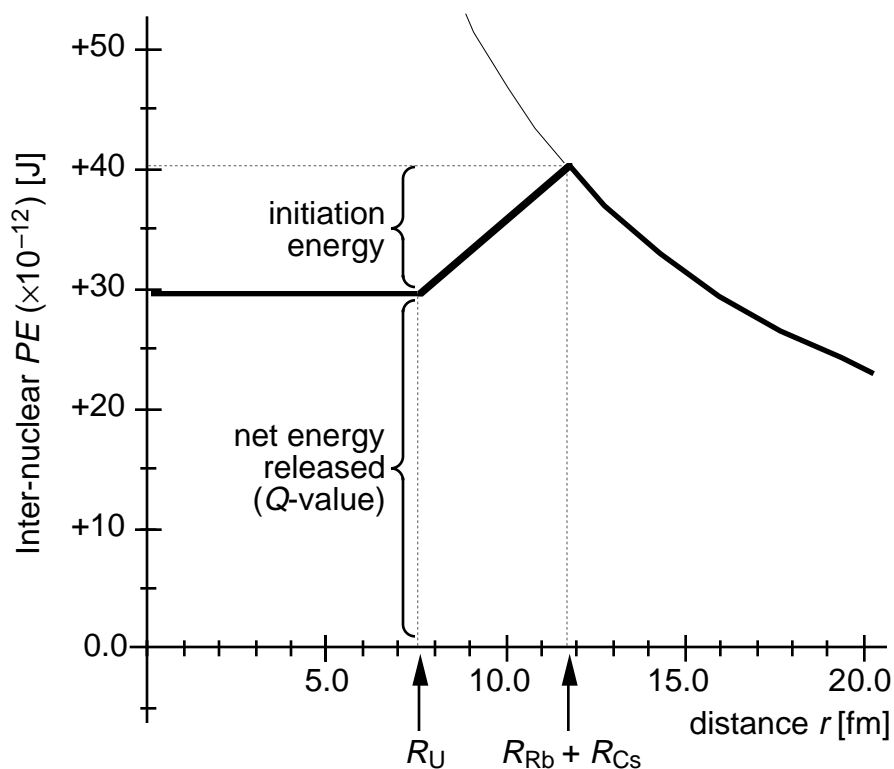
fission

A reaction where a nucleus *splits* into two lighter nuclei (and perhaps other fragments such as neutrons).⁹

Consider a typical fission reaction, first initiated by the absorption of a neutron by a ${}^{235}_{92}\text{U}$ nucleus, which then fragments into ${}^{93}_{37}\text{Rb}$ and ${}^{141}_{55}\text{Cs}$, and two neutrons (which may also trigger other fission reactions in a *chain reaction*):



If we can ignore the reactant neutron and two product neutrons, the **inter-nuclear potential energy** curve, as a function of the separation distance r (measured in **femtometers**) between the centers of the ${}^{93}_{37}\text{Rb}$ and ${}^{141}_{55}\text{Cs}$ nuclei can be drawn as shown below. If the ${}^{235}_{92}\text{U}$ nucleus is in its ground state, then it will have a **nuclear radius** of R_U (as shown in (a) at left). If it absorbs a sufficiently energetic neutron, the total energy of the ${}^{235}_{92}\text{U}$ nucleus will increase



⁹ Note that there is some contention as to whether fission should be strictly defined in terms of a reaction that results in product nuclei with a lower **proton number** Z , resulting in an element further up the periodic table (even though the **nucleon number** A may be higher); or defined in terms of a reaction that results in a nucleus with a lower A , such that it has less nucleons (even though Z may be higher). For the purposes of Block 14 of Physics 7C we will avoid confusing this issue of how the operative term of "lighter" is used in our definition of fission. In any case, the operative term behind the definition of fission should be "splits up."

such that its radius will "elongate" to the point of breaking apart **nuclear strong bonds** (as shown in (b)→(c) on the previous page) after which separated ${}_{37}^{93}\text{Rb}$ and ${}_{55}^{141}\text{Cs}$ nuclei will repel each other electrically (as shown in (d) on the previous page).

(Note that this fission process is *exothermic*, as the final inter-nuclear *PE* is less than the initial inter-nuclear *PE*. If this fission process is run in reverse, it would be an *endothermic fusion* process.) (See **femtometers, fusion, nuclear radius, nuclear strong bonds, nucleon number, potential energy (inter-nuclear), proton number.**)

four fundamental interactions of nature

At the microscopic level, there are only four unique ways that two objects may interact with each other.

Two *charges* may use an electromagnetic interaction in order to exert electric forces on each other (if they are stationary); or magnetic forces on each other (if they are moving). **Electrical potential energy** may be stored in the locations of two charges relative to each other. The range of electromagnetic interactions is infinite.

Two *masses* may use a gravitational interaction in order to exert gravitational forces on each other. **Gravitational potential energy** may be stored in the locations of two masses relative to each other. The range of gravitational interactions is infinite.

Two **nucleons** may use a **nuclear strong interaction** in order to exert strong forces on each other. (The term "strong" being a descriptive term used here for lack of a better choice of words.) **Nuclear strong potential energy** may be stored in the presence (or lack thereof) of a **nuclear strong bond**. The range of nuclear strong interactions is limited to nucleons that are touching (or are very nearly touching, whose surfaces are within one **femtometer** apart).

The **nuclear weak interaction** mediates whether a neutron transforms into a proton, or *vice versa*. (The term "weak" being used here for lack of a better choice of words, and descriptive of its range and effects being shorter and weaker than the nuclear strong interaction.) In this sense the nuclear weak interaction does not behave in much the same way as the other three fundamental interactions of nature described above. The range of weak interactions is limited to a femtometer, within the confines of a neutron or a proton. Additional indications that a nuclear weak interaction has taken place (other than a neutron-proton transformation) is the absorption or emission of an **electron** (or a **positron**), and a **neutrino** (or an **antineutrino**) in the reaction.

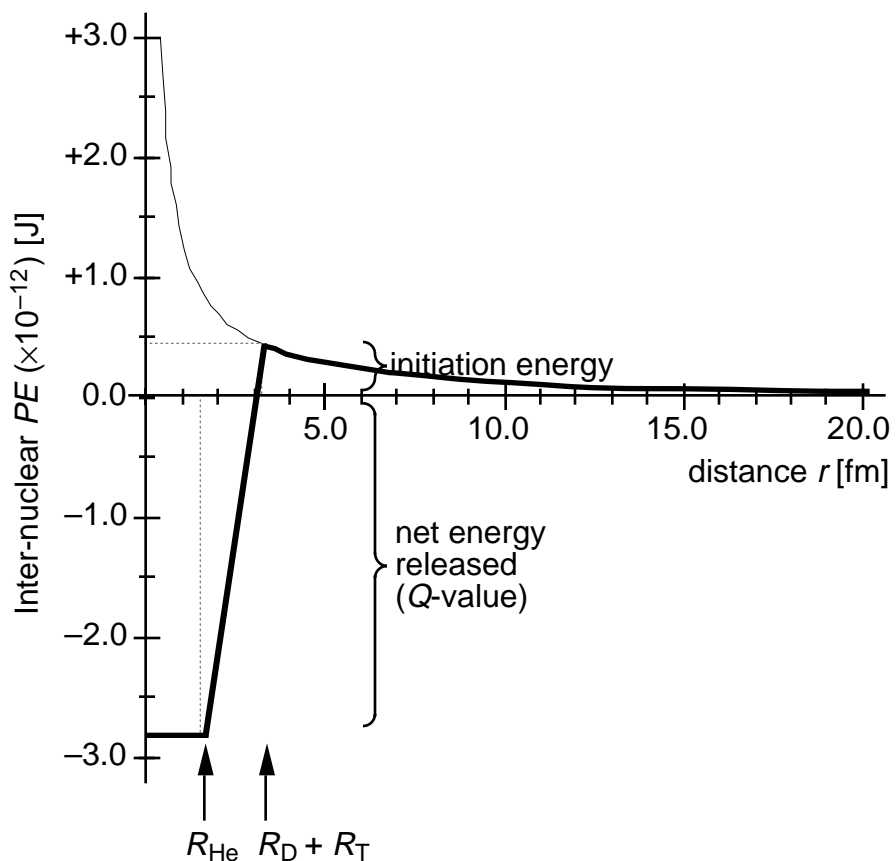
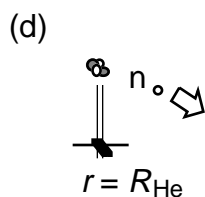
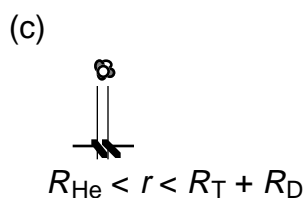
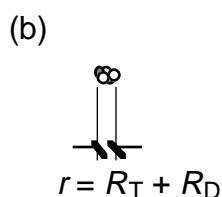
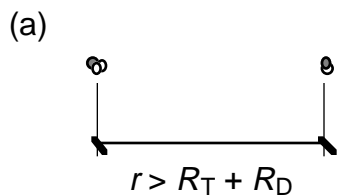
Included in the **inter-nuclear potential energy** is the result of *all* four fundamental forces of nature, although the only

substantive contributions come from electric potential energy and nuclear strong potential energy. (See **antineutrino**, **electron**, **femtometer**, **neutrino**, **nuclear weak interaction**, **nucleon**, **positron**, **potential energy**, **potential energy (inter-atomic)**.)

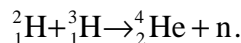
fusion

A reaction where nuclei are *combined* to make a heavier nucleus (which may emit fragments such as protons, neutrons, or gamma rays).¹⁰

Consider a typical fusion reaction, where a *deuterium* nucleus (${}^2_1\text{H}$, sometimes denoted as "D") and a *tritium* nucleus (${}^3_1\text{H}$, sometimes denoted as "T") combine to form a helium (${}^4_2\text{He}$) nucleus (and an ejected neutron):



¹⁰ Note that there is some contention as to whether fusion should be strictly defined in terms of a reaction that results in a product nucleus with a higher **proton number** Z , resulting in an element further down the periodic table (even though the **nucleon number** A may be lower); or defined in terms of a reaction that results in a nucleus with a higher A , such that it has more nucleons (even though Z may be lower). For the purposes of Block 14 of Physics 7C we will try to avoid confusing this issue of how the operative term of "heavier" is used in our definition of fusion. In any case, the key term behind the definition of fusion should be "combined."



If we can ignore the product neutron, the **inter-nuclear potential energy** curve, as a function of the separation distance r (measured in **femtometers**) between the centers of the ${}^2_1\text{H}$ and the ${}^3_1\text{H}$ nuclei can be drawn as shown on the previous page. If the ${}^2_1\text{H}$ and the ${}^3_1\text{H}$ nuclei do not have a sufficient amount of total energy, then they will never come close enough to touch. If they do have enough energy (fusion reactions are typically initiated under extremely high pressures and temperatures), they will come close enough to touch (as shown in (a)→(b) on the previous page) and form **nuclear strong bonds** (as shown in (c) on the previous page), after which they will become a single ${}^4_2\text{He}$ nucleus with a **nuclear radius** of R_{He} (as shown in (d) on the previous page, after emitting an excess neutron).

(Note that this fusion process is *exothermic*, as the final inter-nuclear *PE* is less than the initial inter-nuclear *PE*. If this fusion process is run in reverse, it would be an *exothermic fission* process.) (See **femtometers, fission, nuclear radius, nuclear strong bonds, nucleon number, potential energy (inter-nuclear), proton number.**)

kinetic energy *KE*

The energy stored in the motion of an object. For an object of mass m moving with an initial velocity v_{initial} and then moving with a final velocity v_{final} , its change in kinetic energy is given by:

$$\Delta KE = \frac{1}{2} m \Delta(v^2) = \frac{1}{2} m (v_{\text{final}}^2 - v_{\text{initial}}^2).$$

(Note that for extraordinary velocities (*i.e.*, sizable percentages of the speed of light), *special relativity* is used give a modified expression for the definition of kinetic energy changes. For the purposes of Block 14 in Physics 7C, we will ignore these so-called "relativistic" effects.)

mass decrement

It is interesting to note that the mass of the initial reactant(s) is *not* the same as the mass of the final products in a **nuclear reaction!** This is because of **mass-energy equivalence**, as the energy taken in or given off in a nuclear reaction must have been converted to or from mass, which is "gained" or "lost." *All* reactions involving the transfer of energy in/out of a system have a corresponding mass decrement. However, nuclear reactions are the only type of initial-to-final processes with appreciable amounts of mass decrements, due to

the amount of energy released (large) compared to the masses of the nuclei involved (small).

The mass decrement is given by the difference between the final mass of the products of a nuclear reaction, and the initial mass of the reactant(s) of a nuclear reaction:

$$\text{"mass decrement"} = m_{\text{final}} - m_{\text{initial}}.$$

Note that with this definition, the mass decrement is *negative* for all *exothermic* reactions. The mass decrement is *positive* for all *endothermic* reactions.

The mass decrement of a nuclear reaction is proportional to the **Q-value** of a nuclear reaction.

(Do not confuse the mass decrement of a nuclear reaction with the **mass defect** of a nucleus. They are related quantities, but strictly speaking they are not the same thing.)

(See **mass-energy equivalence, mass defect, nuclear reaction, Q-value.**)

mass defect

It is interesting to note that the mass of a **nucleus** is always *less* than the total mass of its **protons** and **neutrons** added together! This is because assembling a nucleus from its constituent protons and neutrons is an exothermic reaction, and due to **mass-energy equivalence** the energy given off must have been converted from mass, which was "lost" in this assembly process. (In order to *disassemble* a nucleus, one must *add* back in this mass defect in the form of energy, as breaking apart a nucleus is an *endothermic* reaction.) The mass defect is given by the difference between the mass of a nucleus, and the mass of all of its neutrons and protons added together:

$$\text{mass defect} = m_{\text{nucleus}} - m_{\text{protons}} - m_{\text{neutrons}}.$$

Note that with this definition, the mass defect of all nuclei is a *negative* quantity.

Mass defect is related to **mass decrement** but don't confuse these two Δm definitions! One can relate the mass defect of the reactants and products of a nuclear reaction to the mass decrement (and ultimately its endo/exo energetics) of a nuclear reaction. (See **binding energy, mass decrement, mass-energy equivalence, neutron, nucleus, proton.**)

mass-energy equivalence

One of the key results of Albert Einstein's *theory of special relativity* is that mass can "disappear" if it is converted into energy, and energy

can "disappear" if it is converted into mass. Thus mass and energy can be considered as different forms of each other. The quantitative expression that relates the change in total energy of an object compared its Δm change in mass is given by:

$$\Delta(\text{energy}) = (\Delta m)c^2,$$

where $c = 3 \times 10^8$ m/s is the magnitude of the velocity of light in a vacuum.

For the purposes of Block 14 in Physics 7C, we will be concerned with two types of mass changes: **mass decrement** and **mass defect**. The first is the change in mass for an initial-to-final reaction that takes in or gives off energy; the second is the "missing" mass of a nucleus, compared to its separate constituent protons and neutrons. They are respectively proportional to **Q-values** and **binding energies**. (See **binding energy, mass decrement, mass defect, Q-value**.)

"mass number" A

(See **nucleon number**.)

megaelectron-volt (MeV)

A non-standard unit of energy, where $1 \text{ MeV} = 1.0 \times 10^6 \text{ eV}$. (The "mega-" prefix means "million," as a "megabuck" is a million dollars.) Historically, megaelectron-volts are used because they are convenient in describing the energies involved in typical nuclear reactions, which are given in the range of tenths to hundreds of megaelectron-volts of energy. (See **electron-volts**.)

neutron (n)

A subatomic particle with a mass of 1.6750×10^{-27} kg (or 1.008665 u), and zero charge. (The neutron is slightly more massive than the **proton**.) Together with the proton, the neutron is found in nature only within the **nucleus** of an atom. The neutron is denoted by ${}^1_0\text{n}$, or sometimes just "n." (See **proton, nucleus, unified atomic mass unit**.)

neutron number N

The number of **neutrons** inside the **nucleus** of an atom. Thus the neutron number N of a nucleus is the **nucleon number** (A) minus the **proton number** (Z):

$$N = A - Z.$$

(See **neutrons, nucleon, nucleon number, nucleus, proton, proton number**.)

nuclear radius R

Protons and **neutrons** are both **nucleons** that have nearly the same mass, and can be considered to have the same density, and the same radius of 1.2 **femtometers**. For the purposes of Block 14 in Physics 7C, protons and neutrons cannot be compressed, and thus *any* nucleus will have the same density, and a volume that is proportional to the number of nucleons contained in it, and a radius that is proportional to the cubed-root (or one-third exponent) of the number of nucleons contained in it:

$$R = (1.2 \text{ fm})\sqrt[3]{A} = (1.2 \text{ fm})A^{1/3},$$

where A is the **nucleon number** of the nucleus. (See **femtometer**, **neutrons**, **nucleon**, **nucleon number**, **protons**.)

nuclear reaction

A process that rearranges the atomic configuration of nucleons, in order to break apart and/or fuse atomic nuclei. (See **fission**, **fusion**, **chemical reaction**.)

nuclear strong interaction

(See **four fundamental interactions of nature**.)

nuclear strong bond

(See **potential energy**.)

nuclear strong potential energy PE_{strong}

(See **potential energy**.)

nucleon

The particles that comprise the **nucleus** of an atom. Both the **proton** and the **neutron** are considered nucleons, as they have the same density, and essentially have the same mass and radius. (See **neutron**, **nucleus**, **proton**.)

nucleon number A

The number of **nucleons** inside the **nucleus** of an atom. Thus the nucleon number A is the total of the number of **protons** (Z) and the number of **neutrons** (N) in a nucleus. (See **neutron**, **neutron number**, **nucleon**, **nucleus**, **proton**, **proton number**.)

nucleus

The positively charged center of an atom, comprised of **protons** and **neutrons**, surrounded by negatively charged **electrons**.

The total number of nucleons in an atom is called the "**mass number**" or **nucleon number** A . Atoms that have the same number of protons will have the same number of electrons, and thus will have the same chemical properties (which ultimately depend on the configuration of electron orbitals). So all atoms of the same *element* will have the same number of protons, given by its "**atomic number**" or **proton number** Z , even though they may have a different number of neutrons. The full notation for specifying this information about a specific atom is:

$${}^A_Z\text{X} \begin{cases} \text{X} = \text{periodic table name of element,} \\ A = \text{number of neutrons and protons,} \\ Z = \text{number of protons.} \end{cases}$$

For example, ${}^{14}_6\text{C}$ or "carbon-14" is an atom with 14 total nucleons, 6 protons, and $N = (A - Z) = (14 - 6) = 8$ neutrons.

(See **electron, nucleon number, neutron, proton, proton number.**)

partons

Historical term for the particles *inside* **protons** and **neutrons**. Current research has shown that protons and neutrons are made up of partons identified as *quarks* and *gluons*, and current theories predict that these quarks and gluons may themselves be manifestations of incredibly tiny, open-ended or coiled eleven-dimensional *superstrings*. (For the purposes of Block 14 in Physics 7C, we will not be concerned with the fascinating topic of the partons and sub-partons that comprise protons and neutrons.)

potential energy

Since there are four fundamental interactions at the microscopic level (electromagnetic, gravitational, nuclear strong, and nuclear weak), there are four fundamental types of potential energies that can be stored at the microscopic level.

Electric potential energy PE_{elec} is stored in the position of a test charge q , relative to a source charge Q . For a test charge q moved from an initial location $r_{initial}$ to a final location r_{final} , as measured from a source charge Q , its change in electrical potential energy is given by:

$$\Delta PE_{elec} = kQq\Delta\left(\frac{1}{r}\right) = kQq\left(\frac{1}{r_{final}} - \frac{1}{r_{initial}}\right),$$

where the constant $k = 8.99 \times 10^9 \text{ N} \cdot \text{m}^2/\text{Coul}^2$. Electrical potential energy *increases* when like charges are brought closer to each other; or when opposite charges are moved farther away from each other. Electrical potential energy *decreases* when like charges are moved farther away from each other; or when opposite charges are brought closer to each other.

Gravitational potential energy PE_{grav} is stored in the position of a test mass m , relative to a source mass M . For a test mass m moved from an initial location $r_{initial}$ to a final location r_{final} , as measured from a source mass M , its change in gravitational potential energy is given by:

$$\Delta PE_{grav} = -GMm\Delta\left(\frac{1}{r}\right) = -GMm\left(\frac{1}{r_{final}} - \frac{1}{r_{initial}}\right),$$

where the constant $G = 6.67 \times 10^{-11} \text{ N} \cdot \text{m}^2 / \text{kg}^2$. Gravitational potential energy *increases* when masses are moved farther away from each other. Gravitational potential energy *decreases* when masses are brought closer to each other. Usually changes in gravitational potential energy are insignificant on the microscopic level, compared to typical changes in electric, nuclear strong, and nuclear weak potential energies in chemical and nuclear reactions.

Nuclear strong potential energy PE_{strong} is stored in the **nuclear strong force bonds** between **nucleons**:

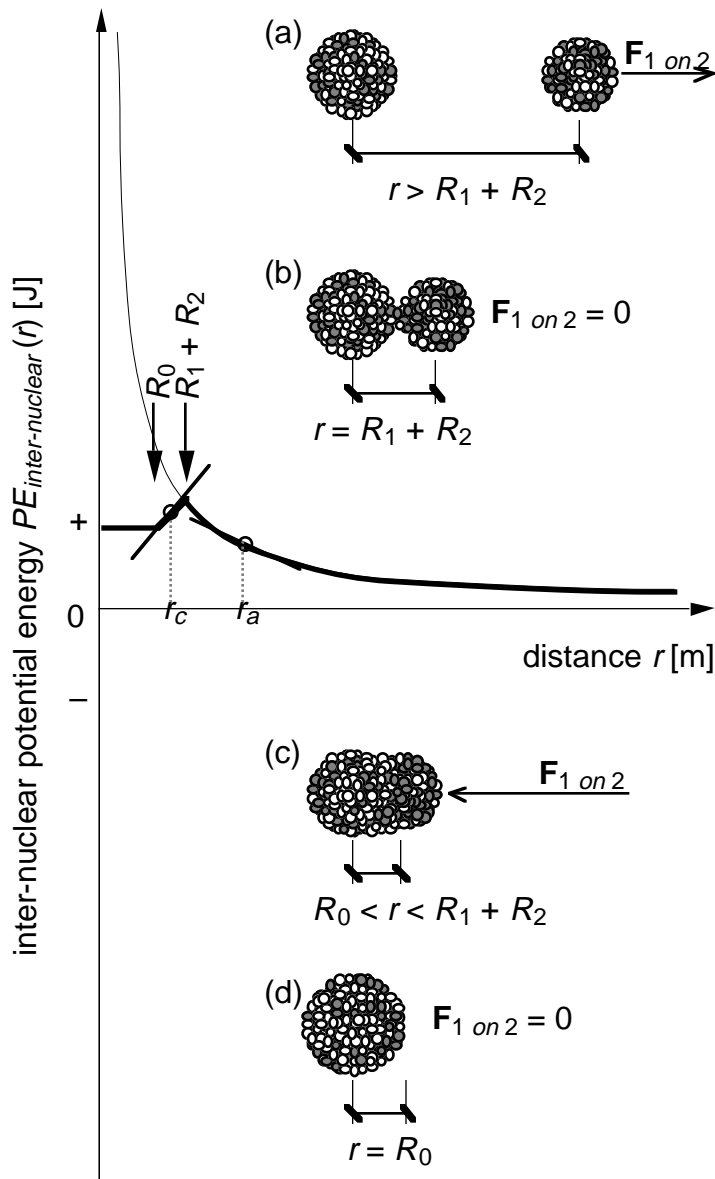
- Strong potential energy *increases* when the strong force bond between nucleons is broken, as when two touching nucleons are separated.
- Strong potential energy *decreases* when a strong force bond between nucleons is made, as when two nucleons are brought together to touch.

Nuclear weak potential energy PE_{weak} is correlated with the mass discrepancy between a neutron and a proton. Since a neutron is slightly heavier than a proton, weak potential energy *decreases* when a neutron transforms into a proton (along with some other particles). Since a proton is slightly lighter than a neutron, weak potential energy *increases* when a proton transforms into a neutron (along with some other particles). (**See four fundamental forces of nature, nuclear weak interaction, nucleon, potential energy (inter-nuclear).**)

potential energy, inter-nuclear

The inter-nuclear potential energy is stored in the position of a test nucleus, relative to a source nucleus, and is the result of *all* four fundamental forces of nature, although the only substantive contributions come from **electric potential energy** and **nuclear strong potential energy**.

The superposition of this far-away electric repulsion and close-together strong force attraction of two nuclei for all r distances between them results in the inter-nuclear potential energy graph shown on the next page, as a function of the center-to-center distance r between the two nuclei. Any and all nucleus-nucleus interactions will have this general shape; to make this graphic specific for a certain type of inter-nuclear interaction only requires specifying the



values $R_1 + R_2$ (the radii of the two nuclei), R_0 (the radius of their combined nucleus) and the values of energy at those locations. Note that the usual convention is to set $PE_{inter-nuclear} = 0$ for $r = \infty$, and to hold the nucleus on the left stationary, while varying the position of the nucleus on the right.

Remember that the **gradient relation** between forces and PE slopes applies here as well. The *magnitudes* of the forces exerted between these interacting atoms can be calculated from the slopes of the $PE_{inter-nuclear}$ graph; the *directions* of the forces involved must point towards decreasing potential energy.

When the nuclei are not in contact with each other (as shown in (a) on the previous page), the only contribution to inter-nuclear potential energy is the electric potential energy of the nuclei exerting repulsive forces on each other, and has a $1/r$ dependence. Bringing the nuclei closer together results in increasing their electric potential energy, and thus increases the inter-nuclear potential energy of this two-nucleus system.

However, if these nuclei are brought close enough to each other, then strong bonds can form between nucleons that touch, releasing energy to the environment. Attractive nuclear strong forces are exerted on the two nuclei (as shown in (c) on the previous page). This reduces the nuclear strong potential energy, and thus also reduces the inter-nuclear potential energy of this two-nucleus system.

Whether the final inter-nuclear potential energy of this two-nucleus system is a positive or negative quantity depends on whether a net amount of energy was taken in or released in combining the two nuclei into a single nucleus. (See **potential energy, Q-value.**)

proton (p)

A subatomic particle with a mass of 1.6726×10^{-27} kg (or 1.007277 u), and a charge of $+1.602 \times 10^{-19}$ Coulombs (which is *exactly* the positive amount of charge of an **electron**). The proton is slightly less massive than the **neutron**. Together with the neutron, the proton is found in nature only within the **nucleus** of an atom. Chemically, a proton should already be familiar to you as the H^+ ion. The proton is denoted by 1_1H , ${}_1^1p$, or sometimes just "p." (See **electron, neutron, nucleus, unified atomic mass unit.**)

proton number Z

The number of **protons** inside the **nucleus** of an atom (also called the **atomic number**), and is synonymous with the element number on the periodic table. *E.g.*, tungsten is element-72, and thus its nucleus has 72 protons.

Q-value

Recall that the difference between the mass of all the initial reactants added together, and the mass of all the final products added together is the **mass decrement** of a nuclear reaction:

$$\text{"mass decrement"} = m_{\text{final}} - m_{\text{initial}}.$$

Due to **mass-energy equivalence**, if the negative of the mass decrement is multiplied by the speed of light squared, one obtains the Q -value of the nuclear reaction—that is, the amount of energy released in a **fission** or **fusion** process:

$$\begin{aligned}\text{"}Q\text{-value"} &= -(\text{mass decrement})c^2, \\ &= (m_{\text{initial}} - m_{\text{final}})c^2,\end{aligned}$$

where $c = 3 \times 10^8$ m/s is the magnitude of the velocity of light in a vacuum. The positive or negative sign of a Q -value will indicate whether a nuclear reaction is exothermic or endothermic:

- If the Q -value is *positive*, the nuclear reaction is *exothermic*. This is because energy was released to the environment, and the Q term is properly on the right-hand (product) side of the nuclear reaction equation.
- If the Q -value is *negative*, the nuclear reaction is *endothermic*. This is because energy from the environment is put into the reaction and the Q term should properly be on the left-hand (reactant) side of the nuclear reaction equation.

For nuclear fusion and fission processes, the problem with evaluating mass decrements and Q -values is that for all physics and chemistry textbooks and references, *masses of nuclei are never given, only atomic masses are given* (in units of **unified atomic mass units**, or u). For the purposes of Block 14 in Physics 7C, we will *not* make the gross approximation that electron masses are negligible (as is done in some physics and chemistry textbooks). Instead, we note that for the majority of fission and fusion processes, the electron masses on both the reactant and product sides of the reaction equation will cancel out, *provided that the atomic masses for hydrogen atoms are used in every instance that lone protons are involved in a nuclear reaction, whether on the left-hand (reactant) side or right-hand (product) side*. Using masses for lone neutrons are okay.

Mass decrement is related to **mass defect** but don't confuse these two Δm definitions! Given the mass defects of each and every

reactant and product nuclei in a reaction, one can determine what the mass decrement of the reaction is, in order to determine whether the reaction is endothermic or exothermic.

$$\begin{aligned}
 \text{"}Q\text{ value"} &= (m_{\text{initial}} - m_{\text{final}})c^2, \\
 &= [(m_{\text{initial}} - m_{\text{protons}} - m_{\text{neutrons}}) - (m_{\text{final}} - m_{\text{protons}} - m_{\text{neutrons}})]c^2, \\
 &= [(\text{mass defect})_{\text{initial}} - (\text{mass defect})_{\text{final}}]c^2, \\
 &= (\text{"binding energy"})_{\text{initial}} - (\text{"binding energy"})_{\text{final}}.
 \end{aligned}$$

Quantitatively the Q -value of a reaction (which determines whether it is endothermic or exothermic) can be found by comparing the total **binding energies** of all the reactants, and the total binding energies of all the products.

Qualitatively a reaction can be determined to be endothermic or exothermic merely by seeing which is larger (less negative)—the binding energy per nucleon of the largest reactant nucleus or the binding energy per nucleon of the largest product nucleus, respectively. (See **binding energy, fission, fusion, mass decrement, mass-energy equivalence, unified atomic mass unit.**)

unified atomic mass unit (u)

A unit of mass, where $1 \text{ u} = 1.66056 \times 10^{-27} \text{ kg}$. The basis for this is arbitrarily defining that the $^{12}_6\text{C}$ atom (presumably because we are carbon-based life forms) has a mass of exactly 12 u.

The unified atomic mass unit (u) replaces the older *atomic mass unit* (amu), which was based on defining the $^{16}_8\text{O}$ atom as having a mass of exactly 16 amu (you'll find this oxygen-based "amu" unit used in older physics and chemistry textbooks). Historically older than the amu is the *dalton*, which was based on defining the hydrogen atom as having a mass of exactly 1 dalton.

For convenience, "blended units" are sometimes used in converting **mass decrements** and **mass defects** expressed in unified atomic mass units to and from MeV:

$$1 \text{ u} = 1.66054 \times 10^{-27} \text{ kg} = 931.48 \frac{\text{MeV}}{c^2},$$

where the c^2 term in the denominator is not explicitly evaluated, but kept merely in order to cancel out other factors of c^2 elsewhere. (See **mass decrement, mass defect.**)

Radioactive decay glossary

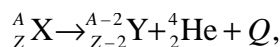
alpha decay	gamma ray (γ)
alpha particle (α)	neutrino (ν)
"antielectron" (e^+)	nuclear weak interaction
antimatter	photon
antineutrino ($\bar{\nu}$)	positron (e^+)
beta decay	potential energy
beta particles (β^- , β^+)	Q-value (radioactive decay)
box model	radioactive decay
electron capture (ϵ)	weak interaction process
gamma decay	

alpha decay

A **radioactive decay** process where a helium nucleus (${}^4_2\text{He}$) is emitted from an unstable proton-rich nucleus, in order for it to reach a more stable configuration by reducing the amount of proton-proton repulsion.¹¹ The helium nucleus emitted from unstable nuclei is referred to as an **alpha particle** (α); in a sense alpha decay is merely a fission process where the daughter nuclei are drastically unequal in size.

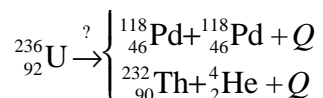
If the resulting nucleus is *still* unstable, it may undergo further alpha decays to remove excess protons, and/or may instead undergo **beta decay** in order to convert excess protons to neutrons. Uranium typically undergoes a long succession of alternating alpha and beta decays (specifically, β^+) to eventually become stable lead.

This process decreases both proton number Z and neutron number N by two (for the remaining nucleus), and is essentially given by:



where "X" and "Y" are generic nuclei, and Q is the energy released for this exothermic process. As with all spontaneous decay processes, alpha decay only occurs if the **Q-value** is positive.

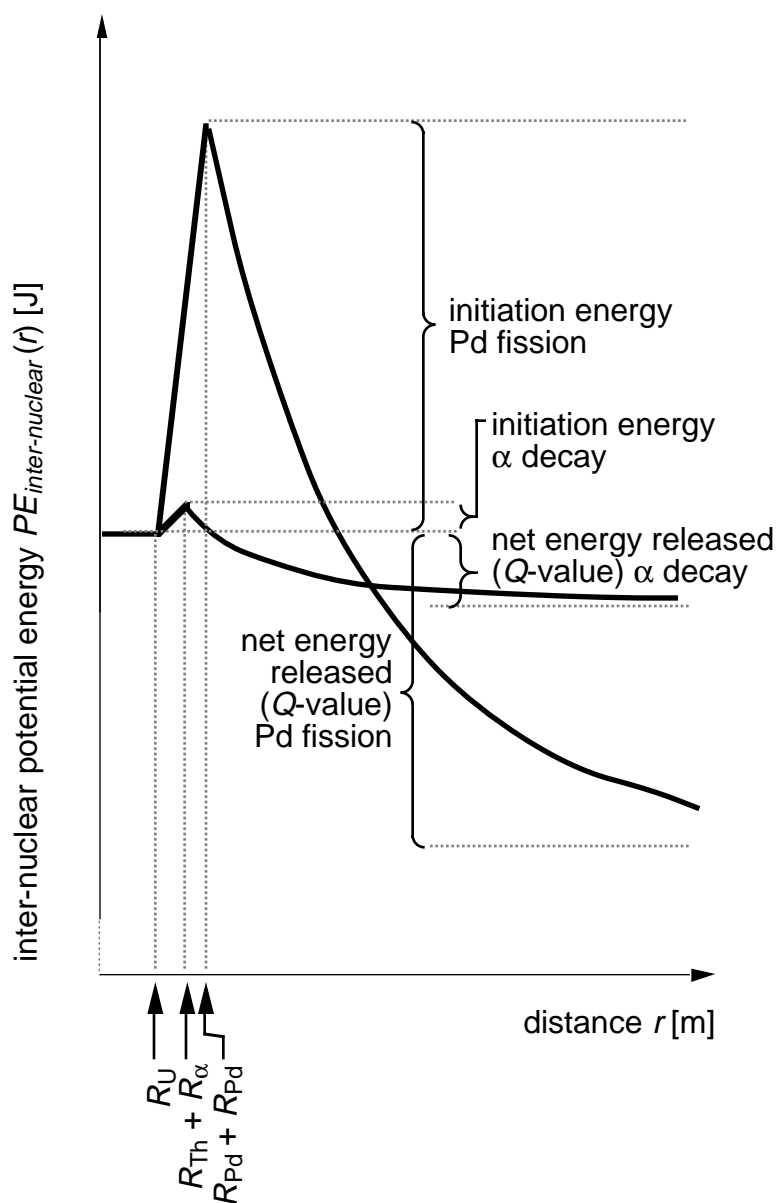
Consider an unstable ${}^{236}_{92}\text{U}$ nucleus, which may either undergo fission into two ${}^{118}_{46}\text{Pd}$ daughter nuclei; or may instead undergo alpha decay to become ${}^{232}_{90}\text{Th}$ in order to stabilize its proton-rich state:



¹¹ All helium on this planet is generated via alpha decay from radioactive elements deep underground (and is eventually removed as an impurity from natural gas wells). Helium atoms in the atmosphere are too light to be gravitationally bound, and eventually escape into outer space!

Both processes result in a release of energy (and thus brings about a more stable final state), but which process will be more likely to occur? By calculating the Q -value for each process, we find that the ${}_{46}^{118}\text{Pd}$ fission process actually releases more energy than alpha decay, but in nature only the alpha emission process is actually observed to occur! So as it turns out for the case of alpha decay, merely evaluating Q -values is insufficient to determine which of two (possible) processes will be more likely to occur (but evaluating Q -values *can* eliminate impossible processes).

In order to understand why alpha decay is the preferred decay process for ${}_{92}^{236}\text{U}$, we can draw their respective **inter-nuclear potential energy** curves, as shown below. Note that fission into



${}_{46}^{118}\text{Pd}$ and ${}_{46}^{118}\text{Pd}$ nuclei would release more energy than the emission of a single alpha particle. However, a much more important factor is the energy cost of initiating either process—the energy cost for initiating fission into two ${}_{46}^{118}\text{Pd}$ nuclei is immense, due to the sheer amount of nuclear strong force bonds that must be broken in order to separate the ${}_{92}^{236}\text{U}$ nucleus into separate ${}_{46}^{118}\text{Pd}$ nuclei, even though once these bonds have broken, there will still be a net energy gain as the two ${}_{46}^{118}\text{Pd}$ nuclei electrically repel each other and lower their potential energy.

In contrast, even though alpha decay will not lower the energy state of the ${}_{92}^{236}\text{U}$ nucleus as much in comparison, there is a lower energy cost for initiating alpha decay, as only a few strong bonds need to be broken in order to separate out a ${}_{2}^{4}\text{He}$ nucleus¹². So alpha decay is the preferred decay process for ${}_{92}^{236}\text{U}$ (which would ultimately reach its preferred lower energy state by a succession of further alpha and beta decays, rather than by a single fission process). (See **alpha particle**, **box model**, **beta decay**, **potential energy (inter-nuclear)**, **Q-value**, **radioactive decay**.)

alpha particle (α)

The historical term for ${}_{2}^{4}\text{He}$, the nucleus of a helium atom, emitted when an unstable nucleus undergoes **alpha decay**. (See **alpha decay**.)

"antielectron" (e^+)

(See **antimatter**, **positron**.)

antimatter

The "evil twin" of matter. Every particle made of matter has an antimatter partner, with exactly the same mass and spin-number, but with an opposite charge (if applicable). The **electron** e^- has an antimatter twin called the **positron** e^+ , the **neutrino** ν has an antimatter twin called the **antineutrino** $\bar{\nu}$. (This list goes on and on; there are antiprotons and antineutrons, *etc.*) Antimatter can be produced in **weak interaction processes**, if there is enough energy transformed into antimatter mass. When an antimatter particle collides with its matter twin, then they annihilate each other and their mass is converted into energy. An in-depth treatment of antimatter and its scientific (and philosophical!) implications is beyond the scope of Block 14 of Physics 7C. (See **radioactive decay**, **weak interaction process**.)

¹² In reality, the actual initiation energy cost is even lower due to the quantum-mechanical *uncertainty principle*, whereby the alpha particle may spontaneously "tunnel" out of the mother nucleus!

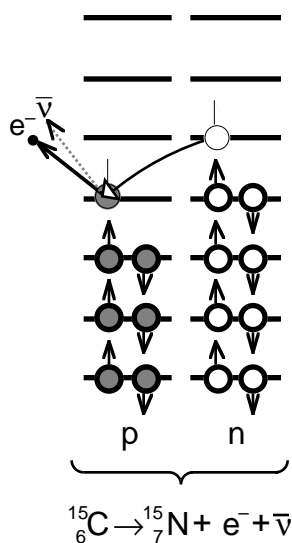
antineutrino ($\bar{\nu}$)

The **antimatter** partner of a **neutrino**. (See **antimatter, neutrino**.)

beta decay

A **radioactive decay** process where either an **electron** or a **positron** is emitted from an unstable nucleus (along with an **antineutrino** or a **neutrino**), in order for it to reach a more stable configuration. Both electrons and positrons are historically referred to as **beta particles** (β^- , β^+ , respectively).

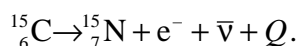
Both β^- and β^+ processes resolve issues with nuclei having too many protons or too many neutrons (compared to its optimal proton-neutron ratio, as discussed in the **box model**), by converting a proton to a neutron, or *vice versa*. Thus beta decays are **weak interaction processes**.



- A β^- decay converts a neutron into a proton, while emitting an electron and an antineutrino, and this exothermic process is essentially given by:



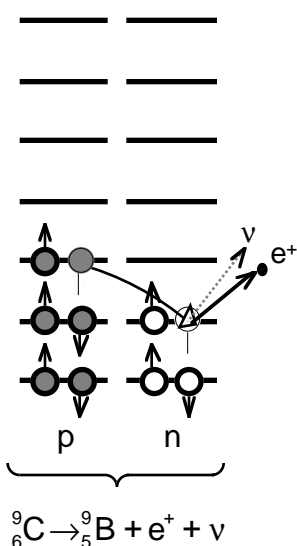
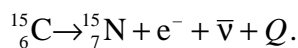
and thus increases proton number Z by one and decreases neutron number N by one. This occurs for nuclei with too many neutrons compared to the number of protons, such that a neutron in a higher energy level can make a transition down to a lower unfilled *proton* energy level, as seen at left for the typical process below:



- A β^+ decay converts a proton into a neutron, while emitting a positron and a neutrino, and this exothermic process is essentially given by:



and thus decreases proton number Z by one and increases neutron number N by one. This occurs for nuclei with too many protons compared to the number of neutrons, such that a proton in a higher energy level can make a transition down to a lower unfilled *neutron* energy level, as seen at left for the typical process below:



Note that a nucleus may also convert a proton to a neutron with an **electron capture** process. (In order to determine which

process will actually convert a proton to a neutron, the **Q-value** of both possible β^+ and electron capture processes must be evaluated.)

As with all spontaneous decay processes, beta decays only occur if the mass of the products is *less* than the mass of the original nucleus, such that their Q -values are positive. (See **antineutrino, box model, beta particles, electron, electron capture, neutrino, positron, Q-value, radioactive decay, weak interaction process.**)

beta particles (β^- , β^+)

The historical term for an **electron** (β^-), or a **positron** (β^+), either of which are emitted when an unstable **nucleus** undergoes **beta decay**. (See **electron, positron, radioactive decay.**)

box model

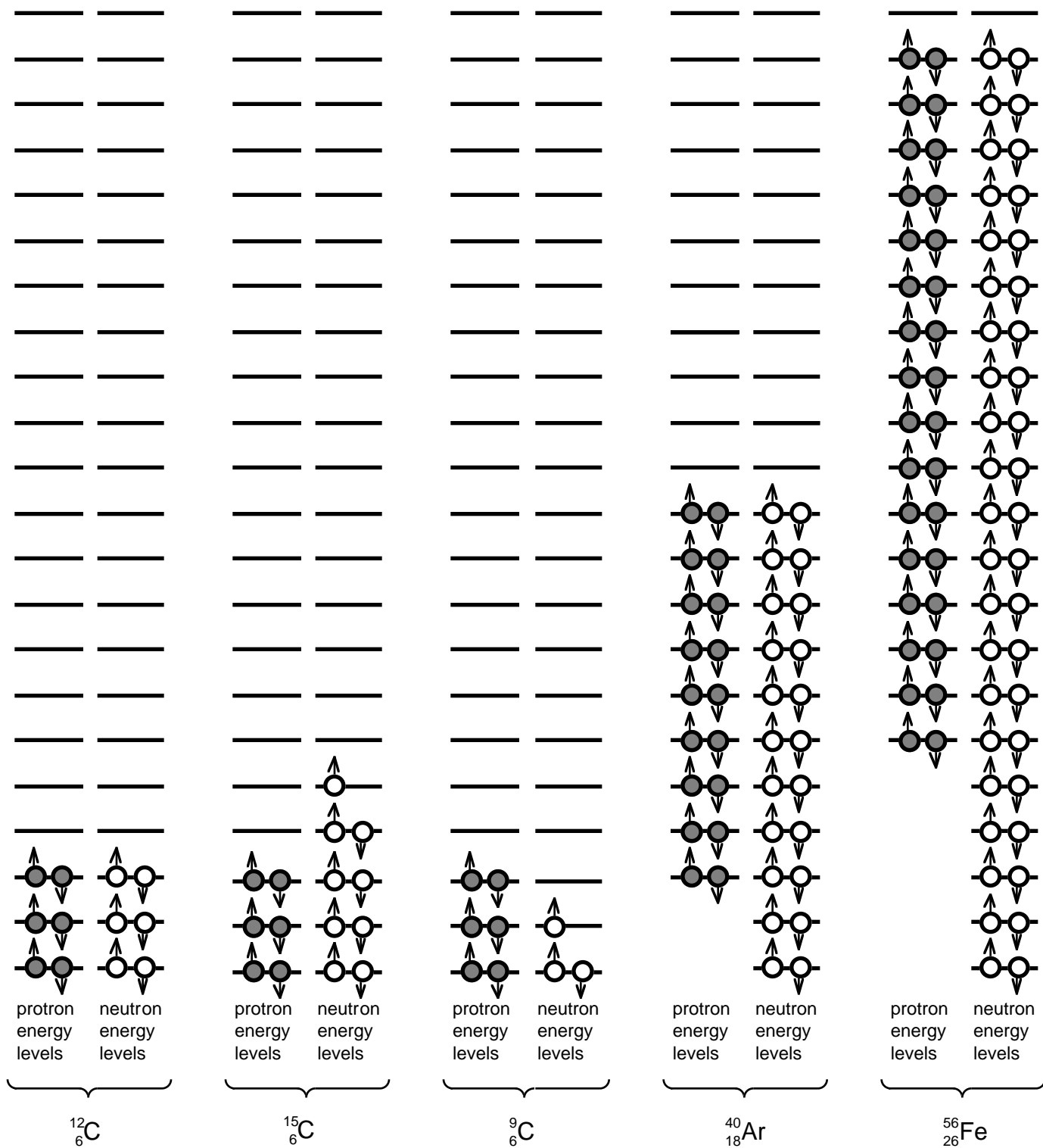
In chemistry, we have seen that an electron that is bound to a nucleus will have quantized energy levels, where a specific amount of energy must be put in for an electron to make a transition from a lower to a higher energy level, and a specific amount of energy will be released when an electron makes a transition from a higher energy level to a lower energy level (usually these energy transfers are in the form of **photons**). Also (at least for the s -orbitals), a maximum of two electrons (spin-up and spin-down) may reside on the same energy level.

We will make a number of analogies between electrons in their atomic orbital energy levels, with protons and neutrons in their nucleus energy levels:

- Since protons and neutrons are both bound to each other *inside* of a nucleus (which is approximated as a "potential box" by nuclear chemists), they will also have quantized energy levels.
- Since protons and neutrons are both "spin-1/2" particles like electrons are, a maximum of two neutrons and two protons (spin-up and spin-down) may reside on the same energy level.

We can depict how protons and neutrons "fill" their respective energy levels in a nucleus, as shown on the next page. Note that for the simplest case of $^{12}_6\text{C}$, there are six protons and six neutrons, and they merely fill up their energy levels from the bottom up, with a maximum of two protons and two neutrons (spin-up and spin-down) per energy level.

For slightly heavier stable nuclei (such as $^{40}_{18}\text{Ar}$), the number of protons is slightly less than the number of neutrons. This is because



all of the protons in a nucleus have mutually repulsive electric forces between them (whereas the neutrons in the nucleus do not have repulsive forces). This causes the ground energy level for the protons to be higher than the ground energy level for the neutrons. When protons and neutrons fill up their energy levels from their ground levels up, this results in slightly less protons than neutrons when the uppermost energy level is filled.

For extremely heavy nuclei (such as ${}^{56}_{26}\text{Fe}$), the number of protons is much less than the number of neutrons. There are so many protons in the nucleus that the mutually repulsive electric forces between them causes the ground energy level for the protons to be shifted up much higher than the ground energy level for the neutrons. This results in much less protons than neutrons in the nucleus when the energy levels are filled from their ground levels up.

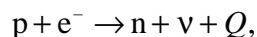
What happens when the uppermost filled proton and neutron energy levels are "uneven?" Then the nucleus is unstable, and will undergo a **radioactive decay** process in order to "equalize" the uppermost filled proton and neutron energy levels! Two unstable isotopes of carbon (${}^{15}_6\text{C}$ and ${}^9_6\text{C}$) are also shown on the next page. (See **photon, radioactive decay**.)

electron capture (ϵ)

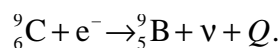
A **radioactive decay** process where an inner orbital **electron** is "captured" by its own unstable nucleus, which subsequently emits a **neutrino**, in order for the nucleus to reach a more stable configuration.

This exothermic process resolves issues with nuclei having too many protons (compared to its optimal proton-neutron ratio, as discussed in the **box model**), by converting a proton to a neutron—thus electron capture is a **weak interaction process**.

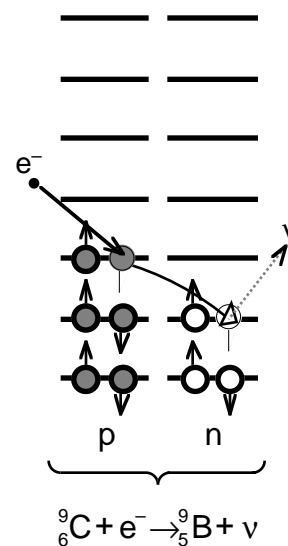
The electron capture process is essentially given by:



and thus decreases proton number Z by one and increases neutron number N by one. This occurs for nuclei with too many protons compared to the number of neutrons, such that a proton in a higher energy level can make a transition down to a lower unfilled *neutron* energy level, as seen at right for the typical process below:



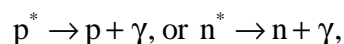
Note that a nucleus may also convert a proton to a neutron with a **beta decay** (specifically, β^+) process. (In order to determine which process will actually occur in converting a proton to a neutron,



the **Q-value** of both possible β^+ and electron capture processes must be evaluated.) (See **beta decay**, **box model**, **electron**, **neutrino**, **Q-value**, **radioactive decay**, **weak interaction process**.)

gamma decay

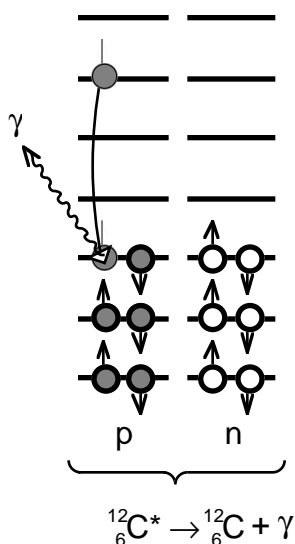
A **radioactive decay** process where an excited (whether stable or unstable) nucleus with a proton or a neutron in a higher energy level very high energy **photon** (in this context historically referred to as a **gamma decay** γ (lower-case Greek letter "gamma") particle), in order to reach a lower energy state. This exothermic process is essentially:



where the asterisk denotes an excited proton or a neutron that is in a higher energy level, as seen in the **box model** representation of nuclear energy levels.

Some things to note for all gamma decay processes:

- A single **neutron** or a single **proton** in a higher energy level makes a transition down to a lower energy level. The initial and final energy levels do not necessarily have to be consecutive levels.
- The number of protons and neutrons in the nucleus remain unchanged. Therefore the proton number Z and neutron number N both remain constant.
- The energy of the photon that is emitted is *exactly* equal to the difference in energy between the levels that the nucleon. Thus it does not make much sense to define a **Q-value** for this process, as when a nuclear reaction for a gamma decay is written out, the energy released in this process is contained in the photon itself.
- "Reverse" gamma decays are possible, where a photon goes into and is absorbed by a nucleus, causing a proton or a neutron to make a transition from a lower energy state to a higher energy state.



In the gamma decay example at left, a proton in a higher energy level makes a transition to the lowest unoccupied energy level, and emits a photon as a result.

By mapping the energies of the photons emitted from gamma decay, the spacings (in MeV) between nuclear energy levels can be deduced.

In the TV show *The Incredible Hulk* (Marvel Productions/Universal TV, 1978-1982), scientist Bruce Banner's genetic structure was mutated by exposure to gamma radiation. (See

box model, gamma ray, neutron, photon, proton, Q-value, radioactive decay.)

gamma ray (γ)

The historical term for a high-energy **photon** emitted when an unstable nucleus undergoes **gamma decay**. (See **gamma decay, photon, radioactive decay**.)

neutrino (ν)

A particle with negligible mass¹³ and zero charge, produced in a typical **weak interaction process**. The neutrino is denoted by ν (the lower-case Greek letter "nu"), while its **antimatter** "twin," the **antineutrino**, is denoted by $\bar{\nu}$ (pronounced "nu-bar"). They are essentially particles that account for energy conservation and *spin* conservation in weak interaction processes, as they don't contribute anything to the mass or the charge of a nuclear reaction (some chemistry textbooks completely neglect the existence of neutrinos and antineutrinos in **beta decay**—give yours a quick read-through!). As such, for our purposes neutrinos and antineutrinos merely act as indicators that a weak interaction process is transforming a neutron into a proton, or *vice versa*.

Nuclear weak force processes in the Sun (where hydrogen protons are converted into neutrons in order to ultimately form helium nuclei) release immense amounts of neutrinos that, because of their neutral charge and essentially zero mass, almost imperceptibly pass through us and the Earth each and every second.

Some physics theories predict that the mass of neutrinos and antineutrinos is not truly zero, but is actually quite (very, very, very) small, and this tiny amount of mass, multiplied by the multitude of neutrinos and antineutrinos in the universe may be the "dark matter" that accounts for the "missing matter" that will be responsible for the ultimate collapse of the universe in a "Big Crunch" tens of billions of years from now. (See **antimatter, beta decay, weak interaction process**.)

nuclear weak interaction

(See **four fundamental interactions of nature, weak interaction process**.)

photon

A transfer of energy (XE) in the form of a light "quantum" either absorbed or transmitted by a quantum system. For a system

¹³ For the purposes of Block 14 in Physics 7C, we will approximate the mass of the neutrino as being zero.

undergoing a transition from an initial energy level $E_{initial}$ to a final energy level E_{final} , the energy contained in a photon is given by:

$$\text{photon } XE = E_{final} - E_{initial}.$$

The sign of the photon XE is (+) when it is absorbed by a quantum system, thus causing an energy level jump. The sign of the photon XE is (−) when it is emitted by a quantum system, due to an energy level fall. (Usually the \pm sign of a photon XE is omitted, as it should be clear whether the photon is absorbed or transmitted by inspection of the specific case being considered.)

The frequency f or wavelength λ of a photon is related to its energy by Planck's constant h :

$$\text{photon } XE = hf_{\text{photon}} = h \left(\frac{c}{\lambda_{\text{photon}}} \right),$$

where $c \equiv 3.00 \times 10^8$ m/s is the velocity of light waves in a vacuum (*i.e.*, the velocity of the photon), and "Planck's constant" $h = 6.626 \times 10^{-34}$ J·s.

A photon can be thought of as particle-like, as the transfer of its energy into or out of a quantum system occurs virtually instantaneously. Thus on the individual photon scale, light behaves like a particle. The absorption or emission of many, many photons simultaneously is indistinguishable from a continuous harmonic wave. Thus on the macroscopic, many photon scale, light behaves like a wave. This is the *wave-particle* duality of light; but in this context this "duality" is merely the interpretation of light in different quantum or macroscopic scales.

Photons are emitted from neutrons and protons making transitions from higher to lower energy levels (as seen in the **box model**) in **gamma decay** processes. (See **box model, gamma decay**.)

positron (e^+)

An **antimatter** subatomic particle with a mass of 9.11×10^{-31} kg (or 0.0005486 u), and a charge of -1.602×10^{-19} Coulombs (which is exactly the positive amount of charge of an **electron**). The positron has *exactly* the same mass as the electron. The positron is usually not found in nature, as when a positron inevitably collides with an electron, they annihilate each other and their combined mass is converted into energy. The positron is denoted by ${}^0_{+1}e$, or sometimes just " e^+ ." (See **antimatter, electron**.)

Q-value (radioactive decay)

Recall that the difference between the mass of all the initial reactants added together, and the mass of all the final products added together is the **mass decrement** of a reaction:

$$\text{"mass decrement"} = m_{\text{final}} - m_{\text{initial}}.$$

Due to **mass-energy equivalence**, if the mass decrement is multiplied by the speed of light squared, one obtains the Q -value of the nuclear reaction—that is, the amount of energy released in a **radioactive decay** process:

$$\begin{aligned} \text{"}Q\text{-value"} &= -(\text{mass decrement})c^2, \\ &= (m_{\text{initial}} - m_{\text{final}})c^2, \end{aligned}$$

where $c = 3 \times 10^8$ m/s is the magnitude of the velocity of light in a vacuum. As *all* radioactive decay processes are *exothermic*, Q -values for these processes are all positive quantities. *Thus if the Q -value for a probable decay process is calculated to be negative, it will be endothermic and thus forbidden.*

As with nuclear fusion and fission processes, the problem with evaluating mass decrements and Q -values for radioactive decay processes is that for all physics and chemistry textbooks and references, *masses of nuclei are never given, only atomic masses are given* (in **unified atomic mass units**, or u). For the purposes of Block 14 in Physics 7C, we will *not* make the gross approximation that electron masses are negligible (as is done in some physics and chemistry textbooks). Instead, we note that for the majority of radioactive decays, the electron masses on both the reactant and product sides of the reaction equation will cancel out. Thus for **alpha decay**, the energy released will be given by the basic mass decrement Q -value equation above (where the final mass includes that of the product nucleus and the **alpha particle**), where *all* these masses are *atomic masses* in the rewritten equation below:

$$\begin{aligned} \text{"}Q\text{-value"} &= (m_{\text{initial nucleus}} - m_{\text{final nucleus}} - m_{\alpha})c^2, \\ &= (m_{\text{initial atom}} - m_{\text{final atom}} - m_{\text{He atom}})c^2. \end{aligned}$$

If the Q -value for a possible alpha decay process is calculated to be negative, it will not spontaneously occur.

For β^- **decay**, the energy released will also be given by the basic mass decrement Q -value equation above, and because *all* these masses are *atomic masses*, conveniently enough the mass of the emitted electron does *not* have to be included (note that the mass of the emitted **antineutrino** is essentially zero):

$$\begin{aligned} \text{"}Q\text{-value"} &= (m_{\text{initial nucleus}} - m_{\text{final nucleus}} - m_e)c^2, \\ &= (m_{\text{initial atom}} - m_{\text{final atom}})c^2. \end{aligned}$$

If the Q -value for a possible β^- decay process is calculated to be negative, it will not spontaneously occur.

A similar convenient simplification happens for **electron capture**—if we *only* use *atomic masses*, then the mass of the captured electron does *not* have to be included (note that the mass of the emitted **neutrino** is essentially zero):

$$\begin{aligned} \text{"}Q\text{-value"} &= (m_{\text{initial nucleus}} + m_e - m_{\text{final nucleus}})c^2, \\ &= (m_{\text{initial atom}} - m_{\text{final atom}})c^2. \end{aligned}$$

For β^+ **decay**, because *all* these masses are *atomic masses*, the mass of the emitted positron complicates the Q -value equation (even though the mass of the emitted **neutrino** is essentially zero):

$$\begin{aligned} \text{"}Q\text{-value"} &= (m_{\text{initial nucleus}} - m_{\text{final nucleus}} - m_e)c^2, \\ &= (m_{\text{initial atom}} - m_{\text{final atom}} - 2m_e)c^2. \end{aligned}$$

Note that *both* electron capture and β^+ decay convert protons to neutrons in unstable proton-rich nuclei. Calculating the Q -value for a proton-to-neutron process will determine which decay process will spontaneously occur:

- If the Q -value for both electron capture and β^+ decay processes are negative, then neither process will spontaneously occur.
- If the Q -value for the electron capture process is positive and for the β^+ decay process is negative, then only electron capture will spontaneously occur.
- If the Q -value for both electron capture and β^+ decay processes are positive, then the β^+ decay process will dominate over electron capture.

Q -values are undefined for **gamma decay** processes, as the energy released in this process is contained in the **gamma ray** photon itself.

(See **alpha particle, alpha decay, antineutrino, beta decay, electron decay, gamma decay, gamma ray, mass decrement, neutrino, unified atomic mass unit.**)

radioactive decay

The historical term for a process that emits (or captures) particles from an unstable nucleus, which results in a more stable nucleus. Radioactive processes encompass **alpha decay**, **beta decay/electron capture**, and **gamma decay**, even though their causes and effects are dramatically different from each other. *All* radioactive decays are *exothermic*, as they are all processes where an unstable nucleus lowers its higher energy state to a relatively more stable lower energy state configuration.

In **alpha decay**, an unstable nucleus emits two protons and two neutrons in the form of a ${}^4_2\text{He}$ nucleus, which is referred to as an **alpha particle** or α .

In **beta decay**, an unstable nucleus either emits an **electron** (also known as a β^- particle), a **positron** (a β^+ particle), or the unstable nucleus "swallows" one of its own inner shell electrons (**electron capture**), all as a means of attempting to become more stable. These three processes are all known as **weak interaction processes**. For all these processes, a neutron is transformed into a proton, or *vice versa*. Also, either an **antineutrino** $\bar{\nu}$ or a **neutrino** ν is emitted.

- For a typical β^- decay, a neutron (either alone, or inside an unstable nucleus) is transformed into a proton, while emitting an electron and an antineutrino. This is how an unstable nucleus with too many neutrons and too little protons can reach a more stable configuration.
- For a typical β^+ decay, a proton inside an unstable nucleus is transformed into a neutron, while emitting a positron and a neutrino. This is how an unstable nucleus with too many protons and too little neutrons can reach a more stable configuration.
- For electron capture, an electron is "swallowed" by a proton inside an unstable nucleus in order to be transformed into a neutron, while emitting a neutrino. (Some textbooks use a lower-case Greek letter ϵ ("eta") to denote electron capture, presumably in order to have a Greek letter for each type of radioactive decay process.) This is another process where an unstable nucleus with too many protons and too little neutrons can reach a more stable configuration, and to find out which proton-to-neutron process will dominate, an analysis of **Q-values** must be performed.

In **gamma decay**, an excited (whether stable or unstable) nucleus emits a very higher energy **photon** (in this context historically referred to as a γ particle), in order for either a neutron or proton in the nucleus to make a transition down to a lower energy level.

Some things to keep in mind for all these forms of radiation emitted by radioactive processes—first, is that they were all labeled well before any knowledge of what actually went on during these nuclear reactions. Also each of these forms of radiation from these exothermic processes carry significant amounts of kinetic energy, which liberate free radicals that can cause genetic damage.¹⁴ It is interesting to note that these three types of radiation are used to treat cancer. Cancerous cells are bad because they divide more often than normal cells¹⁵, but this also makes them much more susceptible than normal cells to radiation damage, as frequent radiation therapy will be most likely to expose cancerous cells that are dividing than normal cells.

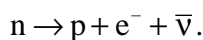
Other radioactive decay processes that you may see mentioned elsewhere are *internal conversion* (where instead of emitting a gamma ray, an excited proton or neutron inside the nucleus directly transfers its excess energy to an inner orbital electron, which is then subsequently freed from its orbital), *proton drip*, and *neutron drip* (where extremely proton-rich and neutron-rich nuclei resort to directly emitting protons and neutrons respectively, instead of converting one into the other inside itself.)

(See **antimatter, electron, neutrino, photon, positron.**)

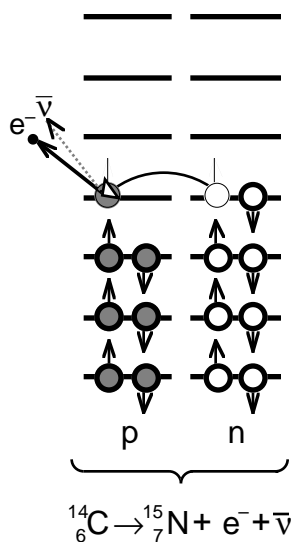
weak interaction process

A **radioactive decay** process where **nuclear weak potential energy** changes, when **neutron** transforms into a **proton**, or *vice versa*, and either an **antineutrino** or **neutrino** is emitted, and an **electron** or **positron** is absorbed or emitted. Examples of weak interaction processes are **beta decays**, and **electron capture**.

Isolated neutrons that are outside of a nucleus have a half-life of about 10 minutes, and will decay into a proton, electron, and an antineutrino:



Since the mass of all the products for this reaction is *less* than the mass of the neutron, then this process is *exothermic*. This is why a neutron may undergo β^{-} decay to make a transition to a proton energy level for the "carbon-14" radioactive decay process shown at

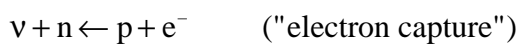
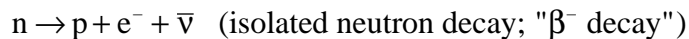


¹⁴ The helium nuclei released from alpha decay are harmless after traveling a few centimeters in air, or after being stopped by a sheet of paper. The electrons and positrons released from beta decay can be harmful if their decay process is energetic enough. The photons released from gamma decay can be very harmful and are very difficult to shield against.

¹⁵ It is remarked that biology is the only science where "divide" means exactly the same thing as "multiply."

left, even if there is negligible difference between the neutron and proton energy levels in the **box model**.

From this fundamental neutron decay all other possible weak interaction processes can be constructed, if we reverse the above process, and conserve charge for particles that are switched from the products to reactants (thus negatively charged electrons "reverse" to positively charged positrons, and antineutrinos "reverse" to become neutrinos, even though antineutrinos and neutrinos are both neutrally charged):



(See **antineutrino, beta decay, box model, electron, electron capture, neutrino, positron, radioactive decay.**)

Block 15 Glossary

Reflection/refraction glossary

critical angle θ_c	refraction
diffuse reflection	reflection
dispersion	Snell's Law
impedance	("Descartes' Law")
index of refraction n	specular reflection
Law of Reflection	surface normal
("Hero's Law")	total internal reflection (TIR)
rays	wheel axle model

critical angle θ_c

When a light ray in a medium with a high **index of refraction** n_1 is transmitted into a medium with a low index of refraction n_2 at a refracted angle of $\theta_2 = 90^\circ$, the incident angle θ_1 is said to be the **critical angle** θ_c . The derivation at right shows how to use **Snell's Law** to solve for the critical angle, given the indices of refraction for the two different media. Remember that an incident ray with an angle θ_1 *equal* to the critical angle θ_c will be transmitted into the n_2 medium with a refracted angle of $\theta_2 = 90^\circ$. This is the idealized boundary case, as seen below. Keep in mind that a critical angle can only be defined for the case where $n_1 > n_2$.

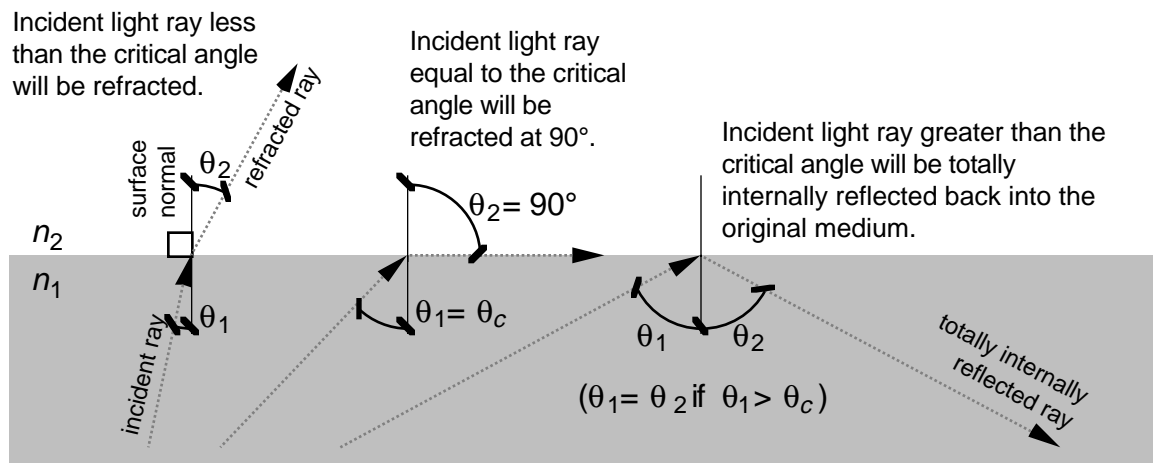
If $n_1 > n_2$:

$$n_1 \sin \theta_c = n_2 \sin 90^\circ,$$

$$\theta_c = \text{Arcsin} \left(\frac{n_2}{n_1} \right)$$

Any incident ray with an angle θ_1 *smaller* than the critical angle θ_c will be transmitted into the n_2 medium, at an angle θ_2 given by **Snell's Law**.

Any incident ray with an angle θ_1 *larger* than the critical angle θ_c will be transmitted back into the original n_1 medium, at an angle θ_2 given by the **Law of Reflection**. In this case, the light ray is said to undergo **total internal reflection** (or **TIR**). (*See critical angle, index of refraction, Law of Reflection, Snell's Law, total internal reflection.*)



diffuse reflection*(See reflection.)***dispersion**

For many materials, the velocity of light through that medium depends on the wavelength of the light wave—velocity is then wavelength-dependent as well as medium-dependent! Thus the **impedance**, and also the **index of refraction** of a medium depends on the wavelength of light that is traveling through the medium. A *dispersive* material has an index of refraction that strongly depends on wavelength; a *non-dispersive* material has an index of refraction that does not depend on wavelength (such that in this ideal case, the wave velocity depends only on the medium).

Rainbows and color aberrations in cheap camera lenses are caused by dispersive materials, such that light of different colors (different wavelengths) refracts differently.

For the purposes of Block 15 in Physics 7C we will be considering only non-dispersive media, unless specifically mentioned otherwise. *(See impedance, index of refraction.)*

impedance

A measure of the "difficulty" a light wave (or any other wave) has in propagating through a medium. The impedance of a medium is inversely correlated with the velocity of light waves through that medium. The impedance of a medium is correlated with bond strengths within (and/or density of) that medium, and proportional to the **index of refraction** of that medium.

index of refraction n

A unitless ratio n measuring "optical density," or how slow light waves travel through a transparent medium, compared to the velocity of light waves in vacuum:

$$n_{\text{medium}} = \frac{c}{v_{\text{medium}}},$$

where $c \equiv 3.00 \times 10^8$ m/s is the velocity of light waves in that medium, and v_{material} is the (slower) velocity of light waves in the transparent medium. Light waves always travel slower through all other media, compared to traveling through a vacuum.

Roughly speaking, the index of refraction of a medium is correlated with the **impedance** of that medium. The index of refraction of a medium is also correlated with the density of (and/or bond strengths within) that medium.

Indices of refraction for several common transparent media are listed at left. (For the purposes of Block 15 in Physics 7C these

Media	Index of refraction n
vacuum	1
air	1.00029
ice	1.31
water	1.329
fused quartz	1.4584
benzene	1.501
Plexiglas™	1.51
crown glass	1.52
zircon	1.923
diamond	2.417

media are all assumed to be relatively **non-dispersive**. Including dispersion effects would mean that the index of refraction for a given material would be wavelength-dependent, instead of being a constant value.) Note that because the velocity of light is slower in all transparent media than through vacuum, all indices of refraction are greater than 1.

E.g., water has an index of refraction of $n = 1.329$, which means that light waves travel 1.329 times *slower* through water than through vacuum. The velocity of light waves through water is given by:

$$v_{\text{water}} = \frac{c}{n_{\text{water}}} = \frac{3.00 \times 10^8 \frac{\text{m}}{\text{s}}}{1.329} = 2.257 \times 10^8 \frac{\text{m}}{\text{s}}.$$

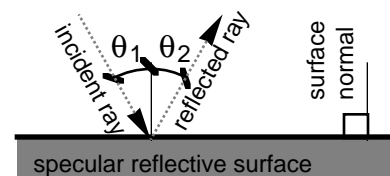
(See **dispersion, impedance**.)

Law of Reflection ("Hero's Law"¹⁶)

The quantitative description of how the θ_1 direction of an incident light **ray** compares to the θ_2 direction of a light ray that undergoes **specular reflection**:

$$\theta_1 = \theta_2,$$

where both these angles are measured "with respect to the **surface normal**." (See **ray, reflection, surface normal**.)



normal

(See **surface normal**.)

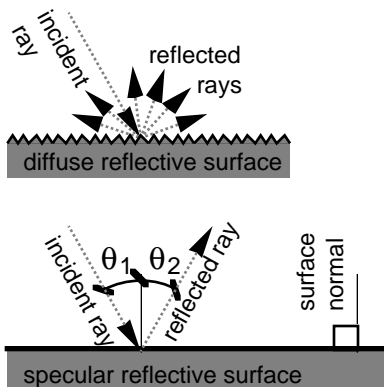
ray

The direction that a light wave travels through a (transparent) medium. A light ray travels in a straight line through a medium, unless it encounters another medium with a different **index of refraction** (or **impedance**). When this happens, the light ray will change direction, either undergoing **reflection** or **refraction**. (See **impedance, index of refraction, reflection, refraction**.)

reflection

When a light **ray** traveling in a certain direction in a transparent medium encounters a solid surface, its direction may change to go back into the original medium. In this case, the light ray is said to be *reflected*. Specifically, there are *two* possible cases how light rays could be reflected back into the original material.

¹⁶ Also known as "Hero's Law," after Alexander of Hero (c. 10-70 A.D.?).



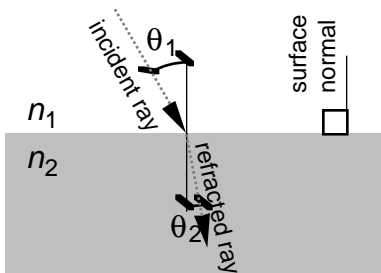
If the surface is not smooth compared to the size of the wavelength of the incident light wave, the light ray will be reflected back in random directions. This is referred to as *diffuse reflection*. Surfaces such as skin, cloth, and paper towels reflect diffusely.

If the surface is smooth compared to the size of the wavelength of the incident light wave, the light ray will be reflected back along a specific direction given by the **Law of Reflection**. This is referred to as *specular reflection*. Mirrors, or very smooth liquid and solid surfaces reflect specularly. While a piece of wood cut by a saw would reflect diffusely, sanding and polishing its surface sufficiently smooth enough (compared to the size of the light wave wavelengths) would cause light rays to reflect specularly! (See **Law of Reflection**.)

refraction

When a light **ray** traveling in a certain direction in a transparent medium (with a certain value of **impedance**, or **index of refraction**) crosses over into another transparent medium (with another value of impedance, or index of refraction), the direction of the transmitted light ray may change. When this happens, the light ray is said to be *refracted*. The quantitative description of how the direction of the light ray changes during refraction is given by **Snell's Law**. (See **Snell's Law**.)

Snell's Law "Descartes' Law"¹⁷

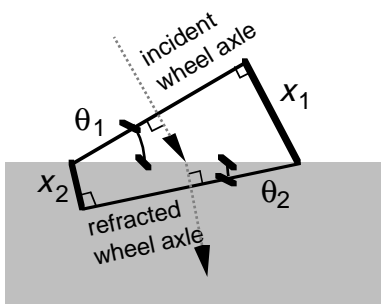


The quantitative description of how the θ_1 direction of an incident light ray (in medium 1) compares to the θ_2 direction of the transmitted light ray (in medium 2) that undergoes **refraction**:

$$n_1 \sin \theta_1 = n_2 \sin \theta_2,$$

where both these angles are measured "with respect to the **surface normal**," and n_1 and n_2 are the respective **indices of refraction** of the incident and transmission media.

Snell's Law may be motivated conceptually by use of the **wheel axle model**, and derived mathematically by noting the different distances Δx_1 and Δx_2 that the light ray (or each wheel) travels in the same Δt time interval, due to the different velocities that the light ray has in either medium. (See **index of refraction, refraction, surface normal, wheel axle model**.)

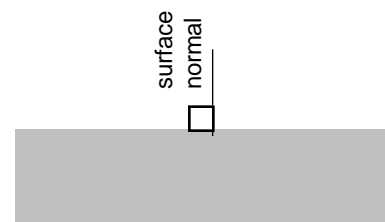


¹⁷

Note that Snell's Law is also referred to as Descartes' Law in France, apparently because that Willebrord Snell (1591-1626) was the Dutch discoverer of this phenomenon, while René Descartes (1596-1650) was the contemporary *French* discoverer of this relation.

specular reflection*(See reflection.)***surface normal (or normal)**

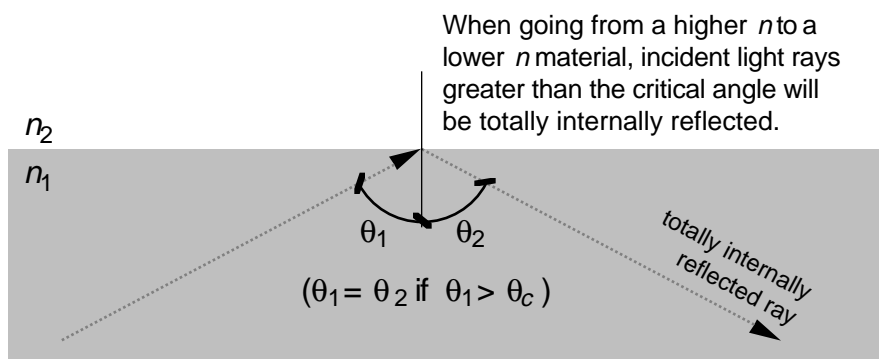
A conceptual line drawn perpendicular a surface, or an interface between two media. The angle of any light **ray** is *always* measured "with respect to the surface normal;" that is, measured between the light ray, and the surface normal. If the interface or surface is curved, then a surface normal is drawn perpendicular to the interface or surface at each and every point. *(See ray.)*

**total internal reflection (TIR)**

There are two criteria that must be satisfied in order for total internal reflection or (TIR) occur:

- (i) A light ray is "attempting" to travel from a medium 1 with a high **index of refraction** n_1 , to a medium with a low index of refraction n_2 . Thus if $n_1 > n_2$, then a **critical angle** θ_c can be defined.
- (ii) The incident ray with an angle θ_1 *larger* than the critical angle θ_c will be transmitted back into the original n_1 medium, at an angle θ_2 given by the **Law of Reflection**. In this case, the light ray is said to undergo total internal reflection (or TIR). *All* of the light in medium 1 is reflected back into medium 2.

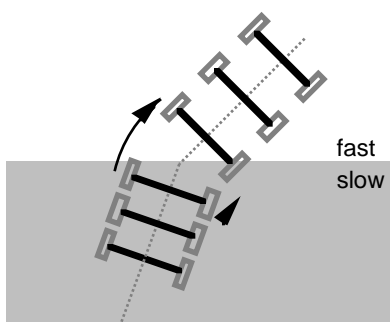
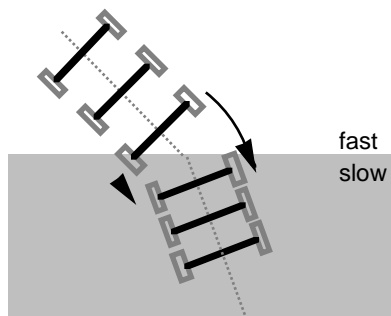
(See index of refraction, critical angle, Snell's Law, Law of Reflection.)

**wheel axle model**

Light may be modeled as literally being a pair of wheels joined together by a common axle. In this wheel axle model, the speed of the wheels depends on the velocity of light in that medium. If the wheel axle encounters an abrupt change in media (with different velocities), each wheel may have a different speed as they change media at different times, such that the direction of the entire wheel

axle may change after both wheels travel in the new medium. Thus the wheel axle model is a means to motivate what happens when a light ray undergoes **refraction**.

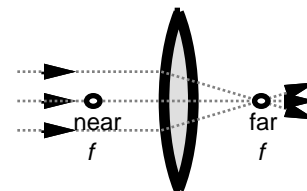
It is important to keep in mind that light is not literally made up of little wheel axles; the justification for thinking that light behaves in this manner is that wheel axles behave just as light would do for abrupt changes in media. Note that more sophisticated models that "explain" the refraction of light are nothing more than just that—models that happen to mimic the behavior of light rays on a more sophisticated level. Some of these models you may encounter in physics textbooks have colorful names such as the *mud marching model*, the *lifeguard model*, *Huygen's wavelet constructions*, the *principle of least time*, or the *principle of least action* (also known as *Feynman path integral formalism*, or *quantum electrodynamics*). Ultimately no one really "knows" why light really does what it does, and why it refracts. The bottom line is that there is a multitude of very sophisticated models that have very elegant means of explaining why light does what it does, and certainly which model is the most thorough, consistent (and preferred) depends on one's tastes (and tolerance for math)! For the purposes of Block 15 in Physics 7C, however artificial and contrived the wheel axle model may be, it suits us very well in *describing* the refraction of light. (See **refraction, Snell's Law**.)



Optical systems glossary**converging lens****diverging lens****far focal point****focal length****focal point f** **image****image distance i** **image height h_i** **lens****magnification, linear M_{linear}** **mirror****near focal point****object****object distance o** **object height h_o** **optical axis****optical system****principal rays****ray tracing****real image****sign conventions****thin lens equation****thin lens model****virtual image****converging lens**

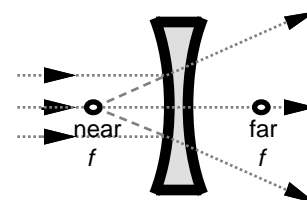
An **optical system** that uses refraction to redirect parallel light rays to converge in towards a **focal point** on the far side of the lens. (After intersecting at the focal point, these rays then diverge outwards.)

A converging lens is also called a *convex lens*, due to the shape of its surfaces. (See **focal point, optical system.**)

**diverging lens**

An **optical system** that uses refraction to redirect parallel light rays to diverge outwards from a **focal point** on the near side of the lens.

A diverging lens is also called a *concave lens*, due to the shape of its surfaces. (See **focal point, optical system.**)

**far focal point**

(See **focal point.**)

focal length f

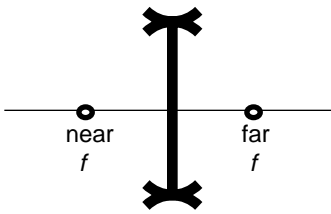
The horizontal distance measured along the **optical axis**, from an **optical system** to a **focal point**:

- The focal length f is defined to be a positive quantity for a **converging lens**.
- The focal length f is defined to be a negative quantity for a **diverging lens**.

(See **converging lens, diverging lens, focal point, optical axis, optical system, thin lens equation.**)

focal point

A point along the **optical axis** of an **optical system**, where parallel light that is redirected by an optical system either converges to, or diverges from:



- The **near focal point** is located on the **object** side of the optical system.
- The **far focal point** is located on the opposite side of the optical system.

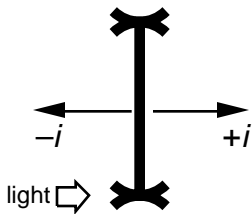
(See **far focal point**, **near focal point**, **object**, **optical axis**, **optical system**, **thin lens equation**.)

image

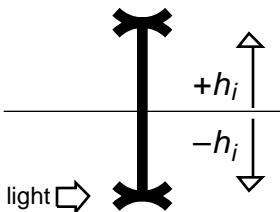
The new location where light rays appear to rediverge from in creating a **real image** or a **virtual image**, after these light rays have been redirected by an **optical system**. (See **optical system**, **real image**, **virtual image**.)

image distance i

Horizontal distance measured along the **optical axis** from the **optical system** to an **image**. For a **lens**, the image distance i is defined to be a positive quantity if it is measured on the opposite side of the lens, with respect to the object. The image distance i is defined to be a negative quantity if it is measured on the same side of the lens as an object. (See **thin lens equation**.)

**image height h_i**

Vertical distance measured from the **optical axis** of an **optical system**, to an **image**. The image height h_i is defined to be a positive quantity if it is measured vertically upwards from the **optical axis**, and a negative quantity if measured vertically downwards from the optical axis. (See **magnification**, **linear**, **thin lens equation**.)

**lens**

An **optical system** that uses refraction to redirect light rays from an **object** in order to create an **image**. (See **converging lens**, **diverging lens**.)

magnification, linear M_{linear}

A ratio of how large (or small) the **image** (measured by its height h_i) is compared to the original **object** (measured by its height h_o):

$$M_{\text{linear}} = \frac{\text{image size}}{\text{object size}} = \frac{h_i}{h_o},$$

$$= -\left(\frac{\text{image distance}}{\text{object distance}}\right) = -\left(\frac{i}{o}\right).$$

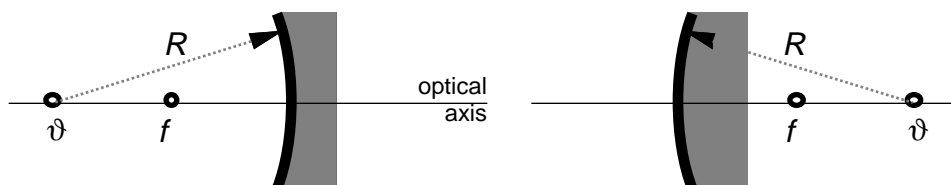
The linear magnification M_{linear} is defined to be a positive quantity if the image is upright or right-side-up (with respect to the object). The linear magnification M_{linear} is defined to be a negative quantity if the image is inverted or upside-down (with respect to the object). *Note that the negative sign is necessary in the $-(i/o)$ form of the linear magnification.*

The absolute value of the linear magnification M_{linear} is less than 1 if the image is smaller than the object. The absolute value of the linear magnification M_{linear} is more than 1 if the image is larger than the object. The absolute value of the linear magnification is equal to 1 if the image is the same size as the object. (See **thin lens equation.**)

mirror

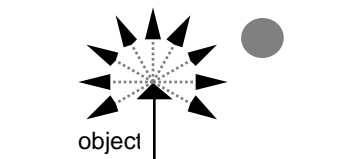
An **optical system** that uses **reflection** to redirect light rays from an **object** in order to create an **image**. A *plane mirror* is a flat reflective surface. A mirror only has one focal point!

Optional to know (but tested for on the MCAT) is that a *converging mirror* is a concave reflective surface (e.g., the inside of a spoon); a *diverging mirror* is a convex reflective surface (e.g., the outside of a spoon). The circular radius R of these curved mirrors is defined to be twice the focal length f of the mirror. The center of these circles is marked by the origin " ϑ " (script upper-case "O"); note that it is twice as far as the focal point away from the surface of a curved mirror.



near focal point

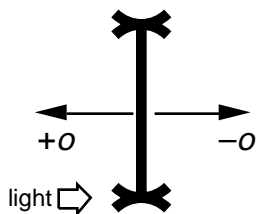
(See **focal point.**)

**object**

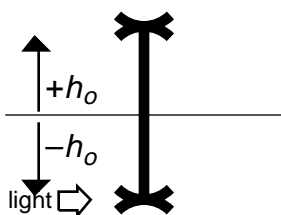
Any source of diverging light rays, idealized as a point source of diverging light rays that may be taken in and redirected by an **optical system** to create an **image**.

object distance o

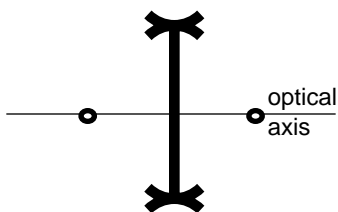
Horizontal distance measured along the **optical axis** from an **object** to the **optical system**. For a **lens**, the object distance o is usually defined to be a positive quantity (some physics textbooks discuss special cases of *virtual objects* which have negative object distances—for the purposes of Block 15 in Physics 7C, we will not be considering those cases). (See **thin lens equation**.)

**object height h_o**

Vertical distance measured from the optical axis of an **optical system**, to an **object**. The object height h_o is defined to be a positive quantity if it is measured vertically upwards from the **optical axis**, and a negative quantity if measured vertically downwards from the optical axis. (See **magnification, linear, thin lens equation**.)

**optical axis**

The (horizontal) line of symmetry drawn perpendicular to a (vertical) **optical system**. The **focal length f** , **object distance o** and **image distance i** are all measured horizontally, along the **optical axis**.

**optical system**

Anything that takes light rays that originally diverged from an **object**, and redirects these light rays (whether by **reflection**, **refraction**, and/or **total internal reflection**) such that they rediverge from a new location, thus creating an **image**.

A *plane mirror* and a *curved mirror* are examples of optical systems that use reflection to redirect light rays.

A *lens* is an optical system that uses refraction to redirect light rays, such as a **converging lens** or a **diverging lens**, or a system of multiple lenses.

A *prism* is an example of an optical system that uses refraction and/or total internal reflection to redirect light rays.

Glasses and *contact lens* are examples of optical systems that use refraction to make light rays diverging from objects that cannot be seen by an eye (due to a congenital or acquired defect in vision) appear to rediverge from an intermediate image that now can be seen by an eye.

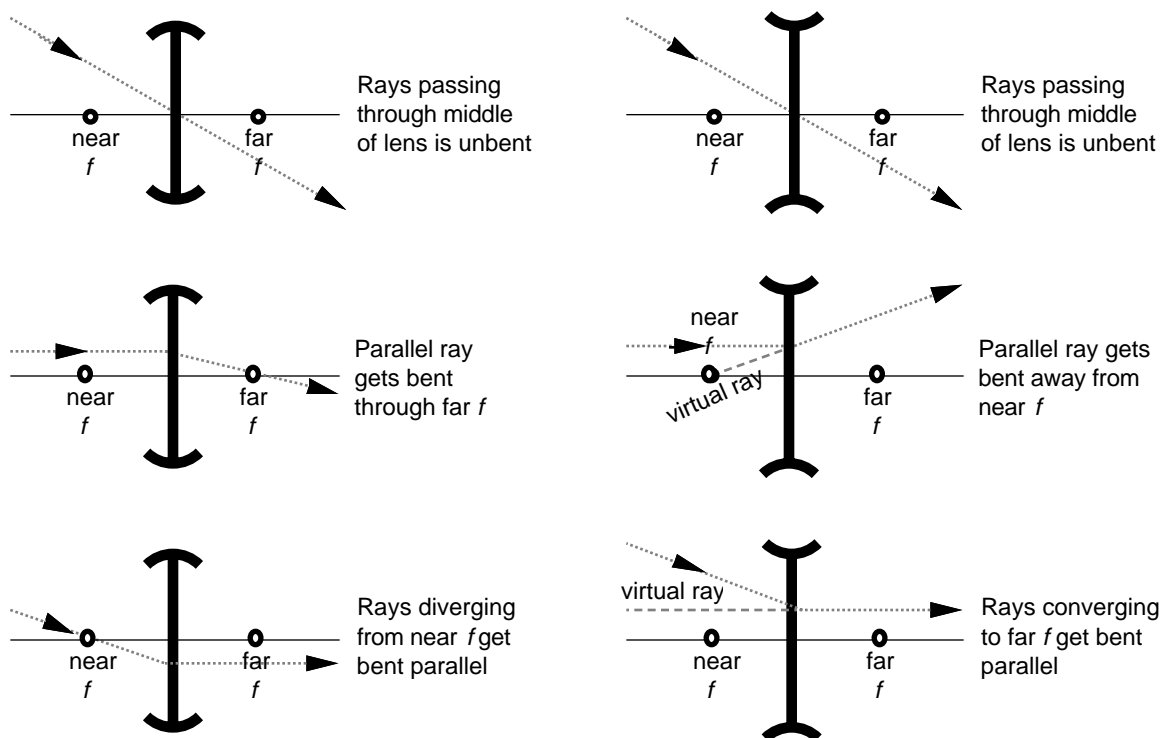
Microscopes and *telescopes* are examples of more complex optical systems that use refraction to make light rays diverging from

objects that "appear small" (whether due to its actual size, or from being far away) appear to rediverge from an image that "appears big."

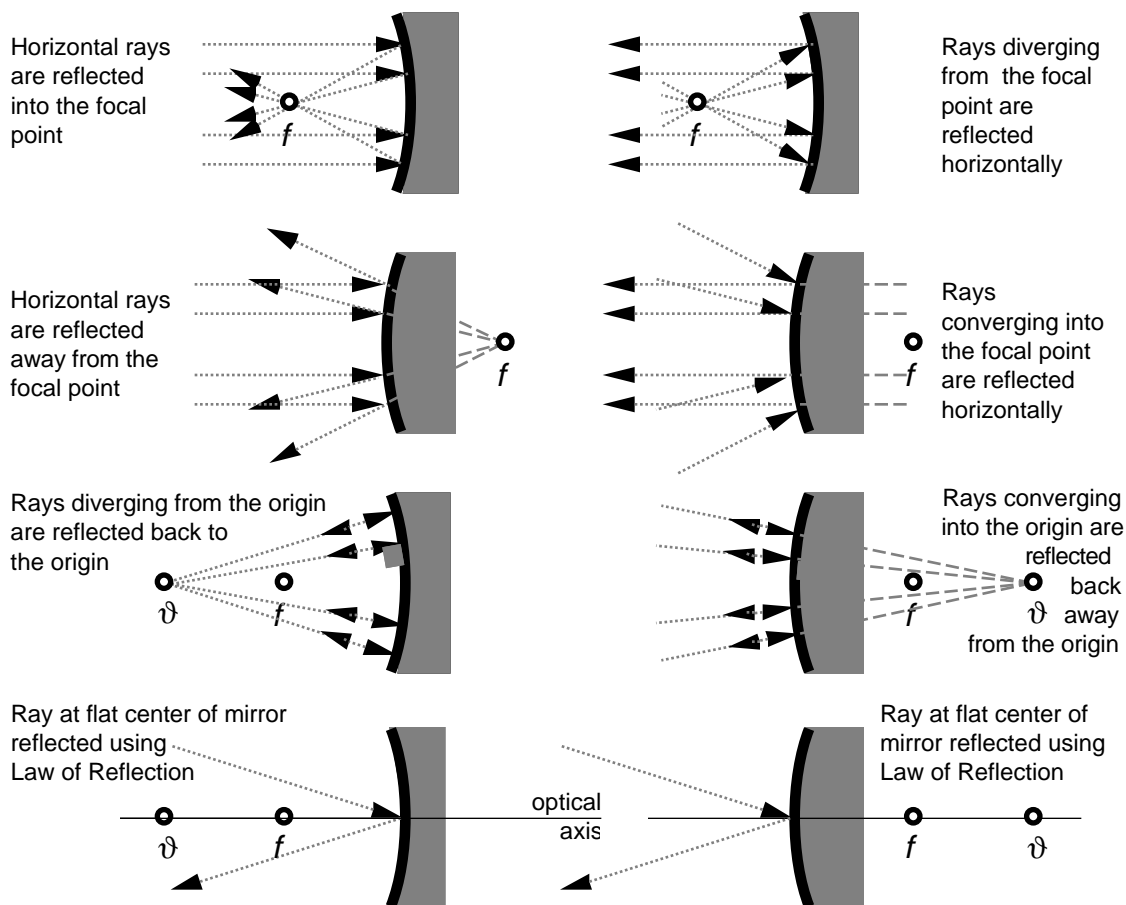
principal rays

Selected light rays from an **object** that an **optical system** redirects to create an **image**. It is important to keep in mind that the principal rays are *not* the only rays that are redirected by an optical system to create an image; the principal rays are merely the simplest redirected rays to draw, in order to geometrically find the location of an image on a **ray tracing**.

There are three principal light rays for a **converging lens**, and three principal light rays for a **diverging lens**.



*Optional to know (but tested for on the MCAT) are the four principal light rays for a **converging mirror**, and four principal light rays for a **diverging mirror**. These principal rays are illustrated here solely for the purposes of completeness in our discussion of optical systems.*



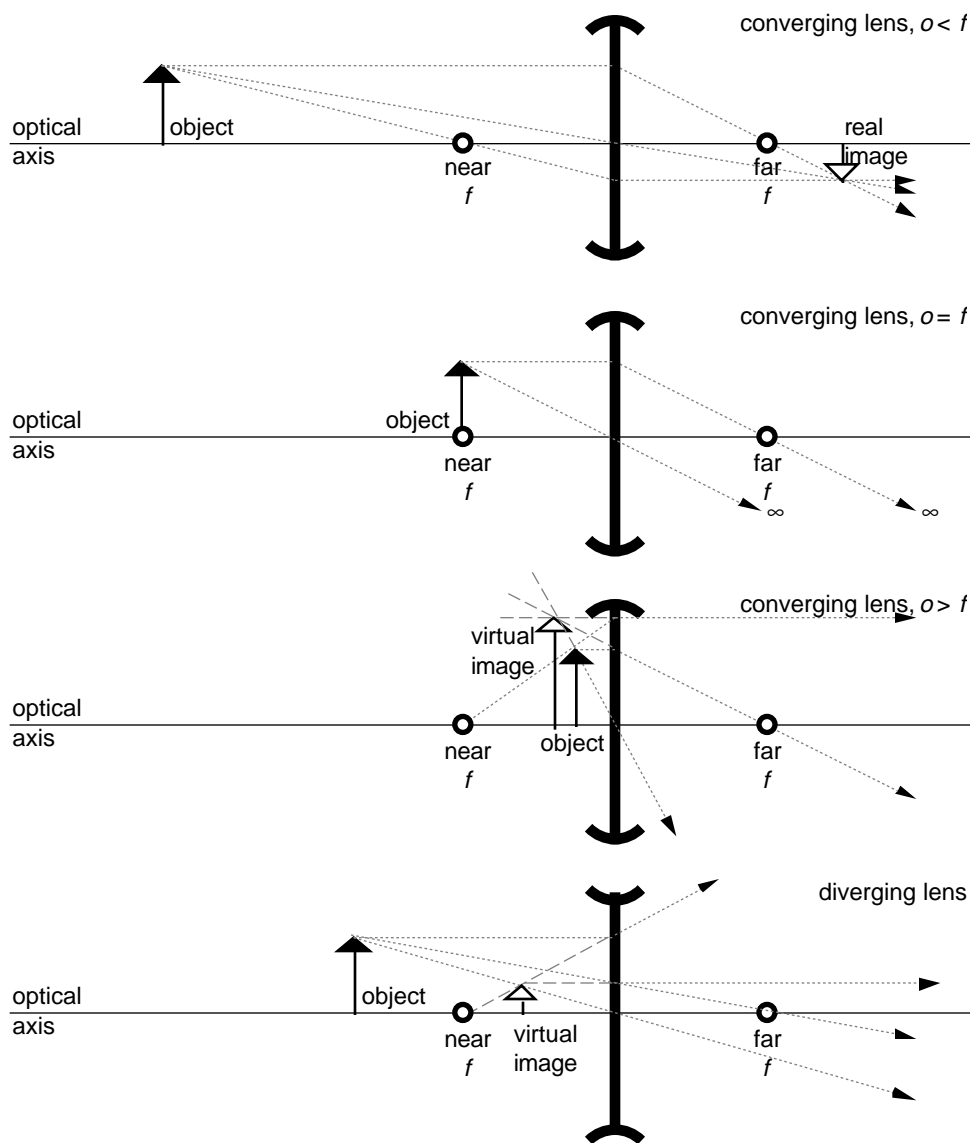
(See **converging lens, diverging lens, ray tracing.**)

ray tracing

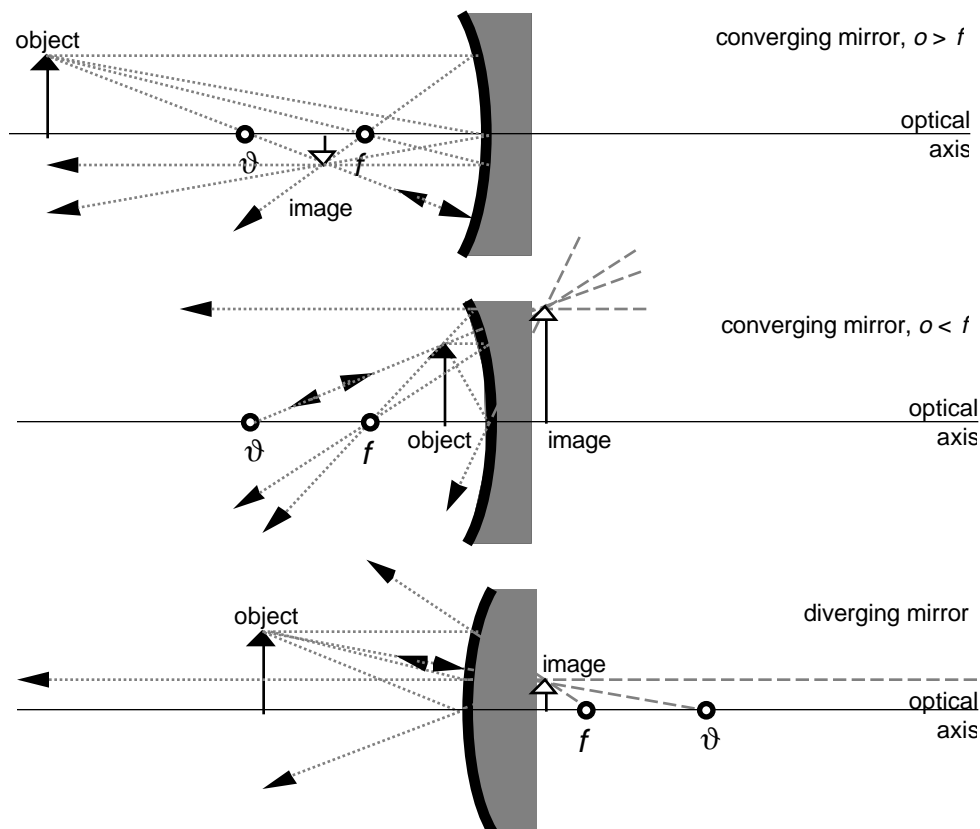
A graphical, geometric method to locate an **image** created by an **optical system** (such as a lens or a mirror) using **principal rays**.

A ray tracing for a **lens** involves drawing light rays that diverge from an object, and located the actual intersection, or the traced-back intersection of light rays to locate the image.

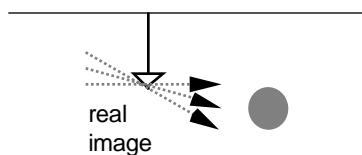
The intersection of the principal rays redirected by an optical system gives the location of the image. Four sample ray tracings for thin lenses are given below. Note that for a simple object in front of a lens, a converging lens has three unique ray tracing results, while a diverging lens produces similar ray tracing results no matter where the object is placed with respect to its near focal point.



Optional to know (but tested for on the MCAT) are three sample ray tracings for thin mirrors are given below. Note that for a simple object in front of a mirror, a converging mirror has two unique ray tracing results, while a diverging mirror always produces the same ray tracing result. These mirror ray tracing diagrams are illustrated here solely for the purposes of completeness in our discussion of optical systems.



(See **converging lens**, **diverging lens**, **principal rays**.)



real image

The location where the light rays redirected by an **optical system** first converge towards, then later diverge from. Thus there is an actual intersection of light rays for a **real image**. (See **ray tracing**, **thin lens equation**.)

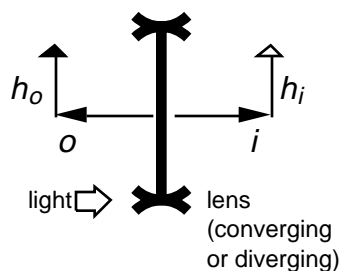
sign conventions

(See **thin lens equation**.)

thin lens equation

A quantitative, algebraic relation between the **object distance** o , **image distance** i , and the **focal length** f of the lens:

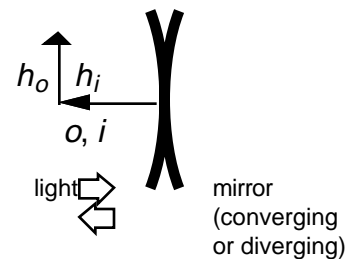
$$\frac{1}{o} + \frac{1}{i} = \frac{1}{f}.$$



The sign convention (\pm) for these parameters is important—the directions of *positive* parameters for a thin lens ($+o$, $+i$, $+h_o$, $+h_i$) are given in the diagram at left, and also discussed in more detail elsewhere.

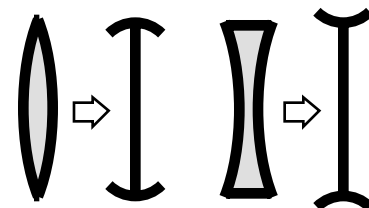
If we know any two of the distances f , o , or i , we can use the thin lens equation to find the third unknown distance. It is always good practice to sketch a ray tracing as well, because it is easy to make mistakes with the positive and negative signs in the thin lens equation.

Optional to know (but tested for on the MCAT) is the *thin mirror equation*, which is actually identical (!) to the thin lens equation, but f and i are defined a little differently. For a thin converging mirror (e.g., the concave inside of a spoon), f is defined to be *positive*. For a thin diverging mirror (e.g., the convex outside of a spoon), f is defined to be *negative*. Since light is reflected back off of mirrors, the *positive* image side of a mirror (converging or diverging) is defined to be the *same* side as the object side. The directions of *positive* parameters for a thin mirror ($+o$, $+i$, $+h_o$, $+h_i$) are given in the diagram at left.



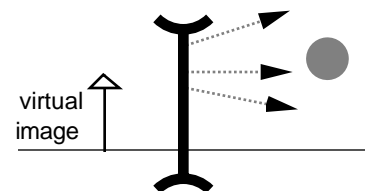
thin lens model

An idealization of a **converging lens** or a **diverging lens** as having negligible thickness, such that the lens redirects all incoming light rays just once. The convention for drawing a thin **converging lens** or a thin **diverging lens** is shown at right. (See **ray tracing, thin lens equation.**)



virtual image

The location where the light rays redirected by an **optical system** appear to diverge from. Thus there is no actual intersection of light rays for a virtual image. (See **ray tracing, thin lens equation.**)



Optometry glossary**accommodation****bifocals****cornea****contact lens****diopter (D)****far point****farsightedness****glasses****hyperopia****multiple lens system****myopia****near point****nearsightedness****optical strength D** **presbyopia****retina****accommodation**

The process by which the ciliary muscles of an eye "squosh" and deform the **cornea** to shorten its **focal length**. This is such that images can be formed on the **retina** of an eye for objects at different distances. The cornea is most deformed in order to focus on objects located at the **near point**.

The ability to accommodate is gradually lost with age, as the cornea eventually becomes aplastic due to **presbyopia**. (See **presbyopia, near point**.)

bifocals

Glasses that have a diverging lens on the top half (to correct for **nearsightedness**), and a converging lens on the bottom half (to correct for **farsightedness**), thus having two different types of focal lengths. It is possible for **contact lenses** to be bifocal as well, when the bottom of the contact lens is weighted.

Trifocals are bifocals with a middle strip that usually is just a flat lens that does not redirect light (for mid-range viewing).

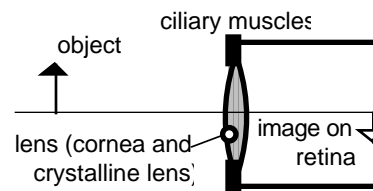
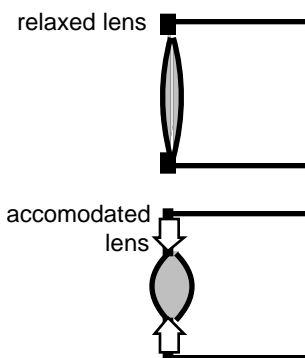
Inevitably, *all* young nearsighted people will be required to wear bifocals to correct for the onset of **presbyopia** in old age. (See **farsightedness, presbyopia, nearsightedness**.)

cornea

The primary optical system of an eye that redirects light rays that diverge from an object, to converge onto the retina to form a real image. The cornea can be idealized as a converging lens with a positive focal length. The focal length of a cornea can be decreased slightly through **accommodation**. (See **accommodation**.)

contact lens

A diverging lens or a converging lens that is placed directly on the **cornea**, in order to correct **nearsightedness** or **farsightedness**, respectively. As it is the first lens in a **multiple lens system**, it takes the light from the original object 1 to produce an intermediate image 1, which is then the object 2 for the cornea (in order to produce a final image 2 on the **retina**). Note that



if the original object 1 is upright, the intermediate image 1 must be upright (and thus a virtual image). This makes the object 2 for the cornea upright, which produces an upside-down final image 2 (which is a real image). (See **multiple lens system**.)

dioptr (D)

Units for the **optical strength** of a lens (such as a contact lens, or a glasses lens). It is defined to be the inverse of the focal length of a lens (which is specified in meters); such that it has units of meters⁻¹ or equivalently, "diopters" or D :

$$\text{optical strength } D[\text{diopters}] = \frac{1}{f[\text{meters}]}$$

If the focal length f of the lens (measured in meters) is positive or negative, then the optical strength D of the lens (measured in diopters) is also correspondingly positive or negative.

far point

The farthest object distance that an eye can focus on. The uncorrected, nominal value for the far point is $+\infty$. A **cornea** is relaxed and does not have to be **accommodated** in order for the eye to see things at the far point (whether or not it is $+\infty$).

If a person's far point is less than $+\infty$, then that person is said to have **nearsightedness** (*i.e.*, cannot see far), or be **myopic**. This condition is not mutually exclusive of **farsightedness**; it is possible for a nearsighted person to be farsighted as well!

farsightedness

(See **near point**.)

glasses

A diverging lens or a converging lens that is placed at a slight distance in front of the **cornea**, in order to correct **nearsightedness** or **farsightedness**. As it is the first lens in a **multiple lens system**, it takes the light from the original object 1 to produce an intermediate image 1, which is then the object 2 for the cornea (in order to produce a final image 2 on the **retina**). Note that if the original object 1 is upright, the intermediate image 1 must be upright (and thus a virtual image). This makes the object 2 for the cornea upright, which produces an upside-down final image 2 (which is a real image).

Note that the space between the glasses lens and a cornea may have a significant effect on how vision defects may be corrected. (See **multiple lens system**.)

hyperopia, hyperopic*(See near point.)***multiple lens systems**

An optical system comprised of more than one lens. For example, a system of two lens can be analyzed by taking things one lens at a time. Light rays from the original object 1 goes through the lens 1, in order to produce an image 1 (whether analyzed using a ray tracing and/or the thin lens equation). This intermediate image 1 is then considered to be the object 2 for lens 2, which produces a final image 2 (whether analyzed using a ray tracing and/or the thin lens equation).

$$\begin{array}{c} \text{object 1} \rightarrow \text{lens 1} \rightarrow \text{image 1} \\ = \\ \text{object 2} \rightarrow \text{lens 2} \rightarrow \text{image 2} \end{array}$$
myopia, myopic*(See far point.)***near point**

The nearest object distance that an eye can focus on. The uncorrected, nominal value for the near point is +25 cm. A **cornea** must be "squoshed" or **accommodated** in order for the eye to see things at the near point (whether or not it is +25 cm).

If a person's far point is congenitally more than +25 cm, then that person is said to have **farsightedness** (*i.e.*, cannot see near), or be **myopic**. This condition is not mutually exclusive of **nearsightedness**; it is possible for a farsighted person to be nearsighted as well!

Children have awesome powers of accommodation, and can "squosh" their corneas in order to attain near points down to +5 to +10 cm.

College students have lost some of their ability to accommodate, and can only partially "squosh" their corneas in order to attain near points of +15 to +20 cm (how do *you* compare?).

Middle-aged adults have lost even more of their ability to accommodate, and can only "squosh" their corneas in order to attain near points of +25 cm. Any more loss of accommodation than this results in **presbyopia** (literally, "elderly eyes"). Even though presbyopia results from the aging process, symptomatically it is indistinguishable from farsightedness, and is corrected in much the same way.

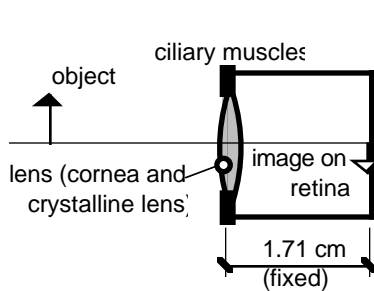
nearsightedness*(See far point.)*

optical strength D

The degree that a lens (such as a contact lens, or a glasses lens) can bend light. A flat piece of glass does not bend parallel light at all, and has a focal length of $f = \infty$, and thus has zero optical strength. This is the motivation behind defining optical strength as being inversely proportional to focal length, such that optical strength has units of meters⁻¹ or equivalently, "diopters." (See **diopter**.)

presbyopia, presbyopic

Literally, "elderly eyes." The loss of the ability to **accommodate**, when the **cornea** becomes aplastic from aging, such that the **near point** increases to more than the nominal value of +25 cm. Symptomatically presbyopia is indistinguishable from **farsightedness**, and is corrected in much the same way. (See **near point**.)



retina

The back surface of the eye, where a real image is formed by the **cornea**. The distance from the cornea to the retina can be idealized as a constant, nominal value of 1.71 cm. Note that even though the real image on the retina is upside-down (!), the visual processes of the brain are conditioned to correct for this unusual circumstance. (It is interesting to note that wearing special inverting goggles will make upright real images on the retina. A volunteer who wore such goggles for several weeks was able to recondition his brain to correct for this, and was fortunately also able to recover after inevitably taking the goggles off at the end of the protracted experiment.)

Physics 7C Quiz Archives

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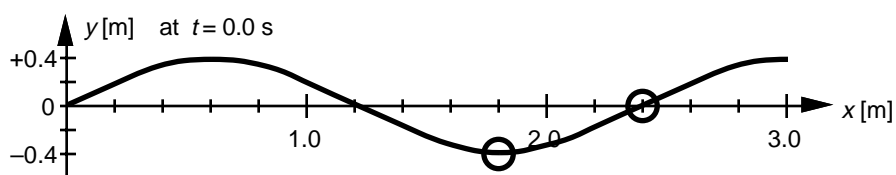
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Quiz 11

1. [40%] An astronaut finds that on Earth a certain pendulum has a period of 1.5 seconds when a 6.9 kg mass is attached and displaced at a maximum angle of 19° . This same astronaut also finds that a certain vertical spring has a period of 2.2 seconds when a 2.0 kg mass is attached and displaced a maximum distance (from its oscillation equilibrium) of 0.28 m.

The astronaut then takes these two experiments with her on her trip to the Moon. This astronaut sends a message back to you on Earth, telling you what the periods of motion are for each of these systems, as they oscillate on the Moon. *Credit is assigned for the completeness and clarity of your justification using the properties of harmonic and wave motion, and not necessarily for stating the correct answers below.*

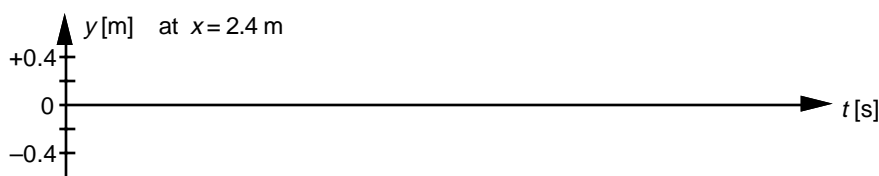
- (a) Explain whether or not you could find the gravitational constant of the Moon (g_{moon}) using the pendulum period on the Moon, and how would you use this information to do so (or why you cannot use this information).
- (b) Explain whether or not you could find the gravitational constant of the Moon (g_{moon}) using the mass-spring period on the Moon, and how would you use this information to do so (or why you cannot use this information).
2. [60%] A transverse wave moves along a horizontal rope at a velocity of 16 m/s, in the positive x direction. A graph of the wave at time $t = 0.0$ s is shown below. *Credit is assigned for the completeness and clarity of your justification using the properties of harmonic and wave motion, and not necessarily for finding the correct answers below.*



- (a) At this very instant in time ($t = 0.0$ s), discuss which rope particle has the faster vertical velocity. *Credit is assigned for the completeness and clarity of your justification using the properties of SHM/wave parameters and harmonic functions, and not necessarily for finding the correct choice below.*

Choose and defend one statement only.

- (A) At $t = 0.0$ s, the particle located at $x = 1.8$ m has a faster vertical velocity than the particle located at $x = 2.4$ m.
- (B) At $t = 0.0$ s, the particle located at $x = 2.4$ m has a faster vertical velocity than the particle located at $x = 1.8$ m.
- (C) At $t = 0.0$ s, both particles have the same vertical velocity.
- (b) For the rope particle located at $x = 2.4$ m, draw a $y(t)$ graph. Be sure to scale and label your horizontal time axes, and plot at least one period of oscillation. You do not need to show your work for this graph, but you need to clearly show your plot.



- (c) If only the properties of the rope medium were changed, and everything else (including the source) remained the same as before, discuss which of the above graphs would change as a result. *Credit is assigned for the completeness and clarity of your justification using the properties of SHM/wave parameters and harmonic functions, and not necessarily for finding the correct choice below.*

Choose and defend one statement only.

- (A) Only the $y(x)$ at $t = 0.0$ s graph will change.
 (B) Only the $y(t)$ at $x = 2.4$ m graph will change.
 (C) Both of the $y(x)$ and $y(t)$ graphs will change.
 (D) Neither of the $y(x)$ and $y(t)$ graphs will change.

Quiz 11 useful equations and constants:

$$y_{SHM}(t) = A \sin\left(\frac{2\pi t}{T} + \Psi_{SHM}\right); \quad y(x,t) = A \sin\left(2\pi \frac{t}{T} \pm 2\pi \frac{x}{\lambda} + \Psi_{wave}\right); \quad \Psi(x,t) = \left(2\pi \frac{t}{T} \pm 2\pi \frac{x}{\lambda} + \Psi_{wave}\right);$$

$$\begin{cases} \frac{d}{dt} A \sin(\beta t) = \beta A \cos(\beta t) \\ \frac{d}{dt} A \cos(\beta t) = -\beta A \sin(\beta t) \end{cases}; \quad T_{pendulum} = 2\pi \sqrt{\frac{L}{g}}; \quad F_{spring} = -k \cdot (\text{stretch}); \quad g_{Earth, at surface} = 9.8 \frac{N}{kg};$$

$$T_{mass-spring} = 2\pi \sqrt{\frac{m}{k}}; \quad f = \frac{1}{T}; \quad \lambda = \frac{v_{wave}}{f}; \quad v_{particle}(x,t) = \frac{d}{dt} y_{particle}(x,t); \quad v_{wave, sound} = \sqrt{\frac{B}{\rho}} \approx 340 \frac{m}{s};$$

$$v_{wave, rope} = \sqrt{\frac{F_{tension}}{\mu}}; \quad v_{wave, light} = \frac{c}{n_{medium}} = \frac{(3.00 \times 10^8 \text{ m/s})}{n_{medium}}; \quad v_{wave, water} = \sqrt{\frac{g\lambda}{2\pi} \tanh\left(2\pi \frac{d}{\lambda}\right)} \approx \sqrt{\frac{g\lambda}{2\pi}}.$$

Quiz 12

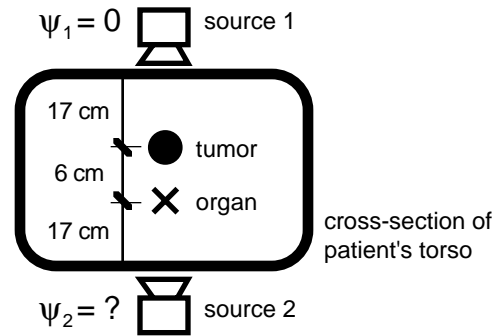
1. [50%] The following two harmonic waves are superposed:

$$y_1 + y_2 = (0.3 \text{ m}) \sin\left(2\pi \frac{t}{(0.12 \text{ s})} - 2\pi \frac{x}{(4.8 \text{ m})} + \frac{\pi}{2}\right) \\ + (0.3 \text{ m}) \sin\left(2\pi \frac{t}{(0.11 \text{ s})} + 2\pi \frac{x}{(4.4 \text{ m})} - \frac{\pi}{2}\right).$$

Credit is assigned for the completeness and clarity of your justification using the properties of wave superposition, and not necessarily for finding the correct answers below.

- (a) What speed and direction is wave 1 traveling? What speed and direction is wave 2 traveling?
- (b) At $x = 0.0$ m, what is the earliest time when constructive interference occurs, after time is started from $t = 0$ seconds?

2. [50%] A proposed new therapy¹⁸ to "cook" and thus destroy a cancer tumor has two microwave emitters that constructively interfere at a tumor (marked with a black dot) inside of a patient's body. There are no reflected microwaves anywhere in the patient's body, and the microwaves have a wavelength of $\lambda = 8$ cm inside human tissue. *Credit is assigned for the completeness and clarity of your justification using the properties of wave superposition, and not necessarily for finding the correct answers below.*



- (a) Determine the *smallest positive value* for the constant phase ψ_2 (in radians) of source 2, if the constant phase ψ_1 of source 1 is zero.
- (b) Using your results for the constant phases of source 1 and source 2 in (a), the patient would like to know whether the microwaves emitted from source 1 and source 2 will leave his vital organ (marked with an "X") intact or not.

Choose and defend one statement only.

- (A) The microwaves emitted from source 1 and source 2 will harm the patient's vital organ.
- (B) The microwaves emitted from source 1 and source 2 will leave the patient's vital organ intact.

Quiz 12 useful equations and constants:

$$y(L, t) = A \sin \left[2\pi \frac{t}{T} - 2\pi \frac{L}{\lambda} + \underbrace{\left(\psi_{\text{source}} + \psi_{\text{reflection}} \right)}_{\psi_{\text{wave}}} \right]; \quad f = \frac{1}{T}; \quad \lambda = \frac{v_{\text{wave}}}{f}; \quad v_{\text{wave, sound}} \approx 340 \frac{\text{m}}{\text{s}} \text{ (air);}$$

$$v_{\text{wave, rope}} = \sqrt{\frac{F_{\text{tension}}}{\mu}}; \quad v_{\text{wave, light}} = \frac{c}{n_{\text{medium}}} = \frac{(3.00 \times 10^8 \text{ m/s})}{n_{\text{medium}}}; \quad v_{\text{wave, water}} = \sqrt{\frac{g\lambda}{2\pi} \tanh\left(2\pi \frac{d}{\lambda}\right)} \approx \sqrt{\frac{g\lambda}{2\pi}};$$

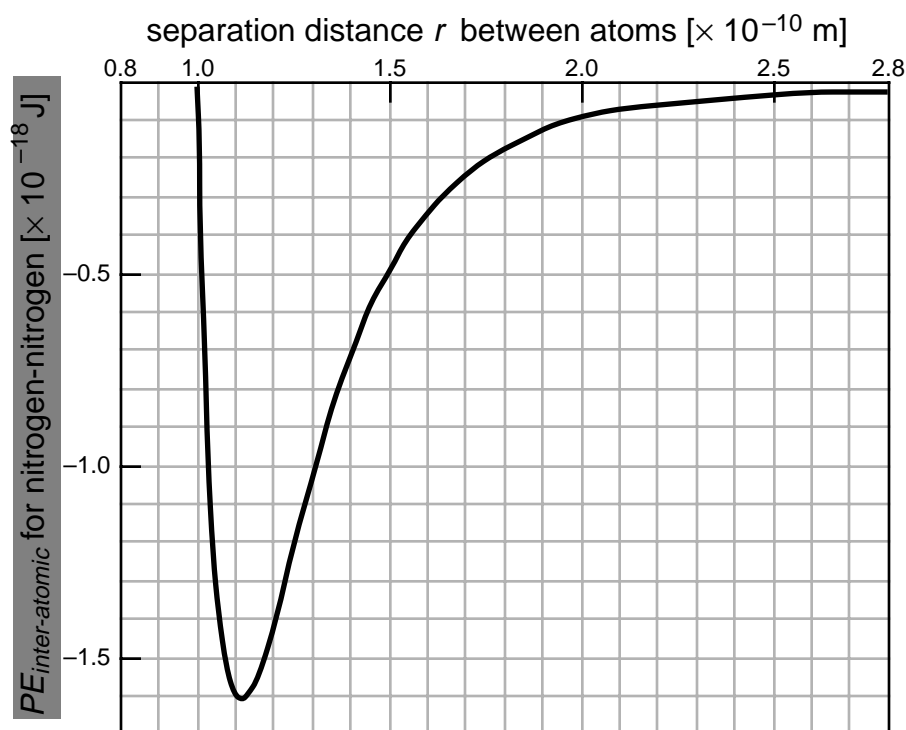
$$\Delta\Psi = \Psi_1 - \Psi_2 = \left(2\pi t(\Delta f) - 2\pi \Delta\left(\frac{L}{\lambda}\right) + \Delta\psi_{\text{sources}} + \Delta\psi_{\text{reflections}} \right); \quad \Delta L = L_1 - L_2;$$

$$\Delta\Psi = \begin{cases} \pm(\text{even})\pi & \text{constructive} \\ \pm(\text{odd})\pi & \text{destructive} \end{cases}; \quad \psi_{\text{reflection}} = \begin{cases} 0 & \text{"soft" reflection;} \\ \pi & \text{"hard" reflection;} \end{cases}; \quad \begin{cases} f_{\text{carrier}} = \frac{1}{2}(f_1 + f_2); \\ f_{\text{beat}} = f_1 - f_2 \end{cases}; \quad \Delta L \approx d \sin\theta.$$

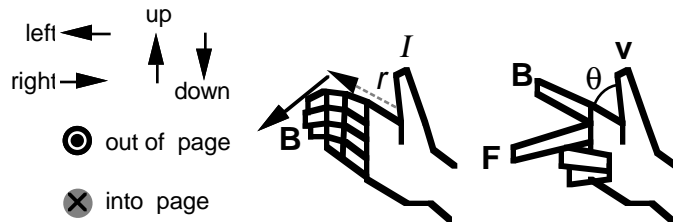
¹⁸ For further information regarding this proposed cancer therapy developed at MIT and licensed by Celsion, Inc., refer to the September/October 2000 issue of Technology Review, p. 25.

Quiz 13

1. [40%] The inter-atomic potential energy as a function of separation distance r of two nitrogen atoms in an N_2 molecule is shown below. *Credit is assigned for the completeness and clarity of your justification using the properties of forces, fields and potential energies, and not necessarily for finding the correct answer.*
- From this graph, determine the separation distance r (to the nearest 0.1 \AA) that corresponds to the *maximum attractive force* between the two nitrogen atoms, and evaluate the magnitude (in N) of this attractive force at this separation distance.
 - Determine the *minimum* amount of total energy (potential plus kinetic) (in J) that this two-atom system must have, in order to attain the maximum amount of attractive force determined in (a).



2. [60%] Shown at right are three parallel wires that are I_1 , I_2 , and I_3 perpendicular to the plane of this page. All three wires contain the same amount of current. In two of the wires, the direction of current is into the page, while in the remaining wire the direction of current is out of the page. The two outermost wires are rigidly held in place. In which direction will the middle wire move? Explain your reasoning using words, pictures, equations and/or calculations. *Credit is assigned for the completeness and clarity of your justification using the properties of forces, fields and potential energies, and not necessarily for finding the correct answer.*



Quiz 13 useful equations and constants:

$$G = 6.67 \times 10^{-11} \frac{\text{N} \cdot \text{m}^2}{\text{kg}^2}; \quad g_M = G \frac{M}{r^2}; \quad g_{\text{Earth, at surface}} = 9.8 \frac{\text{N}}{\text{kg}}; \quad R_{\text{Earth}} = 6.37 \times 10^6 \text{ m};$$

$$M_{\text{Earth}} = 5.98 \times 10^{24} \text{ kg}; \quad \mathbf{F}_{\text{on } m} = m\mathbf{g}; \quad k = 8.99 \times 10^9 \frac{\text{N} \cdot \text{m}^2}{\text{Coul}^2}; \quad E_Q = k \frac{Q}{r^2}; \quad \mathbf{F}_{\text{on } q} = q\mathbf{E};$$

$$q_e = -1.602 \times 10^{-19} \text{ Coul}; \quad \mu_0 = 1.26 \times 10^{-6} \frac{\text{Tesla} \cdot \text{m}}{\text{Amps}}; \quad B_I = \frac{\mu_0 I}{2\pi r}; \quad F_{\text{on } qv} = qvB \sin \theta; \quad F_{\text{along } r} = -\frac{\Delta PE}{\Delta r};$$

$$\Delta PE_{\text{elec}} = kQq\Delta\left(\frac{1}{r}\right); \quad \Delta PE_{\text{grav}} = -GMm\Delta\left(\frac{1}{r}\right) \approx mg\Delta h.$$

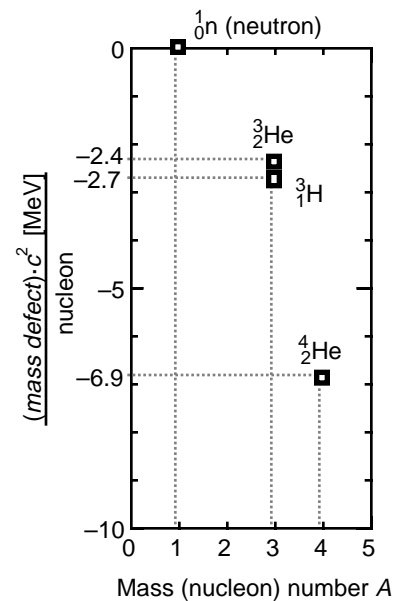
Quiz 14

1. [50%] Consider the values of the $(\text{mass defect}) \cdot c^2 / A$ versus A plot for the various nuclei shown at right.

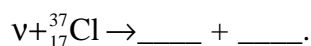
- (a) Which of these three exothermic nuclear reactions will release the most energy to the environment? Choose one of the answers below, and explain your reasoning. Credit is given for your explanation, not necessarily for your choice below. Credit is assigned for the completeness and clarity of your justification using the properties of mass-energy equivalence and nuclear processes, and not necessarily for finding the correct answer below.

Choose and defend one statement only.

- (A) $p + {}^3_1\text{H} \rightarrow {}^4_2\text{He} + \text{energy}$.
 (B) $n + {}^3_2\text{He} \rightarrow {}^4_2\text{He} + \text{energy}$.
 (C) $n + {}^3_2\text{He} \rightarrow {}^3_1\text{H} + p + \text{energy}$.
- (b) As shown on the plot above, an isolated neutron has a $(\text{mass defect})/A$ value of zero. Explain why this is so. Credit is assigned for the completeness and clarity of your justification using the properties of mass-energy equivalence and nuclear processes.



2. [50%] The "Super Kamiokande" detector in Japan uses a massive underground tank of carbon tetrachloride to absorb and detect the rate of neutrinos emitted from the Sun that pass through the Earth.
- (a) [15%] If a ${}^{37}_{17}\text{Cl}$ nucleus in this tank absorbs a neutrino, complete the rest of the nuclear reaction below. *Credit is assigned for the completeness and clarity of your justification using the properties of mass-energy equivalence and nuclear processes, and not necessarily for finding the correct answer.*



- (b) [35%] Find the (minimum) amount of energy (in either J or MeV) that will either be released or taken in by this reaction, and clearly state whether this reaction will be exothermic/endothermic. Given below is a table of atomic (*not* nuclear masses) for ${}^{37}_{17}\text{Cl}$ and five possible product nuclei, in unified atomic mass units. *Credit is assigned for the completeness and clarity of your justification using the properties of mass-energy equivalence and nuclear processes, and not necessarily for finding the correct answer and exothermic/endothermic choice.*

Table of atomic masses:

$${}^{37}_{17}\text{Cl} = 36.965\,902\,600\text{ u}$$

$${}^{38}_{17}\text{Cl} = 37.968\,010\,550\text{ u}$$

$${}^{37}_{16}\text{S} = 36.971\,125\,716\text{ u}$$

$${}^{38}_{18}\text{Ar} = 37.962\,732\,161\text{ u}$$

$${}^{37}_{18}\text{Ar} = 36.966\,775\,912\text{ u}$$

$${}^{36}_{18}\text{Ar} = 35.967\,546\,282\text{ u}$$

Useful equations and constants:

$${}^A_{Z(\text{protons})}\text{X}(\text{element}); R = (1.2\text{ fm})A^{1/3} = (1.2 \times 10^{-15}\text{ m})A^{1/3}; \text{ "Binding energy" } = (\text{mass defect})c^2;$$

$$\text{ "Q - value" } = (\text{mass decrement})c^2; XE_{\text{photon}} = hf_{\text{photon}} = \frac{hc}{\lambda_{\text{photon}}}; h = 6.626 \times 10^{-34}\text{ J}\cdot\text{s};$$

$$n \rightarrow p + e^- + \bar{\nu}; \Delta PE_{\text{elec}} = kQq\Delta\left(\frac{1}{r}\right); q_e = -1.602 \times 10^{-19}\text{ Coul}; 8.99 \times 10^9 \frac{\text{N}\cdot\text{m}^2}{\text{Coul}^2};$$

$$\left\{ \begin{array}{l} m_{\text{electron}} = 9.10939 \times 10^{-31}\text{ kg} = 5.4858 \times 10^{-4}\text{ u} \\ m_{\text{proton}} = 1.672623 \times 10^{-27}\text{ kg} = 1.007276\text{ u} \\ m_{\text{neutron}} = 1.674929 \times 10^{-27}\text{ kg} = 1.008665\text{ u} \end{array} \right\}; \begin{array}{l} 1\text{ u} = 1.66054 \times 10^{-27}\text{ kg} = 1.49242 \times 10^{-10} \frac{\text{J}}{\text{c}^2}; \\ c = 3 \times 10^8\text{ m/s}; 1\text{ eV} = 1.602 \times 10^{-19}\text{ J}; \\ 1\text{ MeV} = 1 \times 10^6\text{ eV}. \end{array}$$

Final Exam formula sheet (comprises Quizzes 11-14, plus additional Block 15 information):

$$y_{SHM}(t) = A \sin\left(\frac{2\pi t}{T} + \Psi_{SHM}\right); \quad y(x,t) = A \sin\left(2\pi \frac{t}{T} \pm 2\pi \frac{x}{\lambda} + \Psi_{wave}\right); \quad \begin{cases} \frac{d}{dt} A \sin(\beta t) = \beta A \cos(\beta t) \\ \frac{d}{dt} A \cos(\beta t) = -\beta A \sin(\beta t) \end{cases};$$

$$T_{pendulum} = 2\pi \sqrt{\frac{L}{g}}; \quad F_{spring} = -k \cdot (\text{stretch}); \quad g_{Earth, at surface} = 9.8 \frac{N}{kg}; \quad T_{mass-spring} = 2\pi \sqrt{\frac{m}{k}}; \quad f = \frac{1}{T};$$

$$\lambda = \frac{v_{wave}}{f}; \quad v_{particle}(x,t) = \frac{d}{dt} y_{particle}(x,t); \quad v_{wave, sound} \approx 340 \frac{m}{s}; \quad v_{wave, light} = \frac{c}{n_{medium}} = \frac{(3.00 \times 10^8 \text{ m/s})}{n_{medium}}.$$

$$y(L,t) = A \sin \Psi = A \sin \left[2\pi \frac{t}{T} - 2\pi \frac{L}{\lambda} + \underbrace{\left(\Psi_{source} + \Psi_{reflection} \right)}_{\Psi_{wave}} \right]; \quad \Delta L = L_1 - L_2; \quad \Delta L \approx d \sin \theta;$$

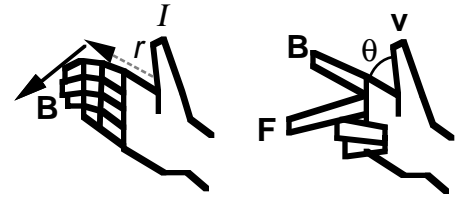
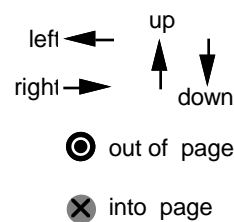
$$\Psi_{reflection} = \begin{cases} 0 \text{ "soft" reflection;} \\ \pi \text{ "hard" reflection} \end{cases}; \quad \begin{cases} f_{carrier} = \frac{1}{2}(f_1 + f_2), \\ f_{beat} = f_1 - f_2 \end{cases};$$

$$\Delta \Psi = \Psi_1 - \Psi_2 = \left(2\pi t(\Delta f) - 2\pi \Delta \left(\frac{L}{\lambda} \right) + \Delta \Psi_{sources} + \Delta \Psi_{reflections} \right) = \begin{cases} \pm(\text{even})\pi \text{ constructive} \\ \pm(\text{odd})\pi \text{ destructive} \end{cases}.$$

$$G = 6.67 \times 10^{-11} \frac{N \cdot m^2}{kg^2}; \quad g_M = G \frac{M}{r^2};$$

$$g_{Earth, at surface} = 9.8 \frac{N}{kg}; \quad R_{Earth} = 6.37 \times 10^6 \text{ m};$$

$$M_{Earth} = 5.98 \times 10^{24} \text{ kg}; \quad \mathbf{F}_{on m} = m\mathbf{g};$$



$$k = 8.99 \times 10^9 \frac{N \cdot m^2}{Coul^2}; \quad E_Q = k \frac{Q}{r^2}; \quad \mathbf{F}_{on q} = q\mathbf{E}; \quad q_e = -1.602 \times 10^{-19} \text{ Coul}; \quad \mu_0 = 1.26 \times 10^{-6} \frac{\text{Tesla} \cdot m}{\text{Amps}};$$

$$B_I = \frac{\mu_0 I}{2\pi r}; \quad F_{on qv} = qvB \sin \theta; \quad F_{along r} = -\frac{\Delta PE}{\Delta r}; \quad \Delta PE_{elec} = kQq\Delta \left(\frac{1}{r} \right); \quad \Delta PE_{grav} = -GMm\Delta \left(\frac{1}{r} \right) \approx mg\Delta h;$$

$$\frac{A(\text{nucleons})}{Z(\text{protons})} X(\text{element}); \quad R = (1.2 \text{ fm}) A^{1/3} = (1.2 \times 10^{-15} \text{ m}) A^{1/3}; \quad \text{"Binding energy"} = (\text{mass defect})c^2;$$

$$\text{"Q-value"} = (\text{mass decrement})c^2; \quad XE_{photon} = hf_{photon} = \frac{hc}{\lambda_{photon}}; \quad h = 6.626 \times 10^{-34} \text{ J}\cdot\text{s}; \quad n \rightarrow p + e^- + \bar{\nu};$$

$$\begin{cases} m_{electron} = 9.10939 \times 10^{-31} \text{ kg} = 5.4858 \times 10^{-4} \text{ u} \\ m_{proton} = 1.672623 \times 10^{-27} \text{ kg} = 1.007276 \text{ u} \\ m_{neutron} = 1.674929 \times 10^{-27} \text{ kg} = 1.008665 \text{ u} \end{cases} \quad \begin{cases} 1 \text{ u} = 1.66054 \times 10^{-27} \text{ kg} = 1.49242 \times 10^{-10} \frac{\text{J}}{c^2} \\ c = 3 \times 10^8 \text{ m/s}; \quad 1 \text{ eV} = 1.602 \times 10^{-19} \text{ J}; \\ 1 \text{ MeV} = 1 \times 10^6 \text{ eV}. \end{cases}$$

$$\theta_1 = \theta_2; \quad n_1 \sin \theta_1 = n_2 \sin \theta_2; \quad \frac{1}{o} + \frac{1}{i} = \frac{1}{f}; \quad M = \frac{h_i}{h_o} = -\left(\frac{i}{o} \right); \quad \begin{cases} n_{vacuum} = 1. \\ n_{water} = 1.33 \end{cases}; \quad D = \frac{1}{f}; \quad \begin{cases} o_{near} = 0.250 \text{ m} \\ o_{far} = \infty \end{cases}$$

Epilogue

*We shall not cease from exploration
And the end of all our exploring
Will be to arrive where we started
And know the place for the first time.
Through the unknown, remembered gate
When the last of earth left to discover
Is that which was the beginning;
At the source of the longest river
The voice of the hidden waterfall
And the children in the apple-tree
Not known, because not looked for*

*But heard, half-heard, in the stillness
Between two waves of the sea.
Quick now, here, now, always—
A condition of complete simplicity
(Costing not less than everything)
And all shall be well and
All manner of thing shall be well
When the tongues of flame are in-folded
Into the crowned knot of fire
And the fire and the rose are one.*

—T. S. Eliot
Excerpt from "Four Quartets"