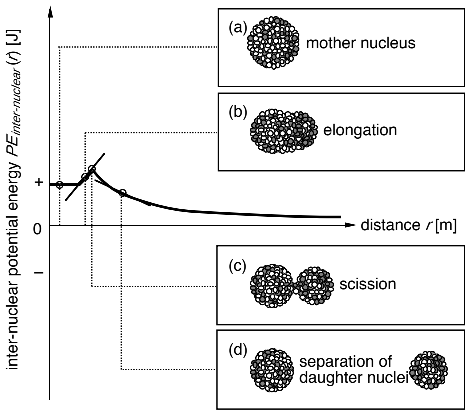
Inter-Nuclear *PE* Graphs: Beyond Balancing Reaction Equations and Calculating Mass Defects in Analyzing Fusion, Fission, and α Decay



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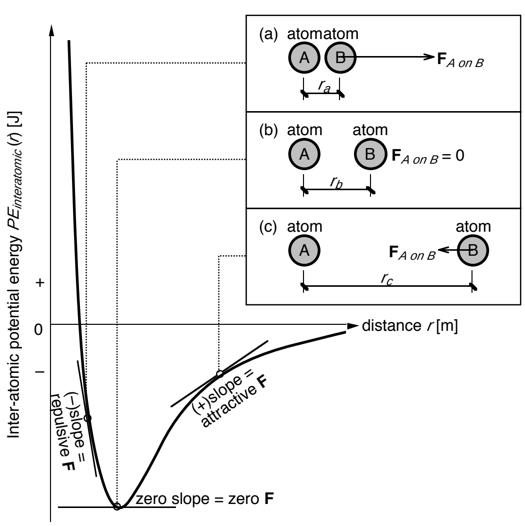
Outline

- Motivation for internuclear *PE* graphs
  - Connection with interatomic *PE* graphs
  - Characterizing certain nuclear processes
- Construction of internuclear *PE* graphs
  - General features
  - Exothermic fission example
  - Exothermic fusion example
- Analysis of nuclear processes
  - Fusion/fission initiation energy paradox
  - α decay versus "symmetric fission" paradox
- Conclusions
  - So what can internuclear *PE* graphs do?

• Motivation for internuclear *PE* graphs
Connection with interatomic *PE* graphs

Interatomic *PE* graphs
- A.k.a. "Lennard-Jones" plots
- Characterizes atom-atom interactions
- *PE* gradient (slope) = force



- Similarly, an internuclear *PE* graph can be constructed to characterize nuclear-nuclear interactions

• Motivation for internuclear *PE* graphs
Characterizing certain nuclear processes

Usual textbook coverage of various nuclear processes:

Fusion, Fission, α decay, $\beta^{\pm}$ (ε) decay, γ decay }
- Balancing reaction equations
- Exo/endo- energetics from mass decrements
- Exponential decay of unstable nuclei

Emphasis on describing "what happens," not explaining "why!"

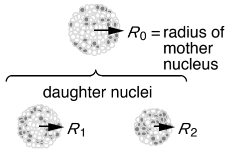
Two "toy" models can motivate the "why" of nuclear processes!

Fusion, Fission, α decay } "Internuclear *PE* graphs" (discussed here)
N. Bohr and J. A. Wheeler, *Phys. Rev.* **56**, 426 (1939)

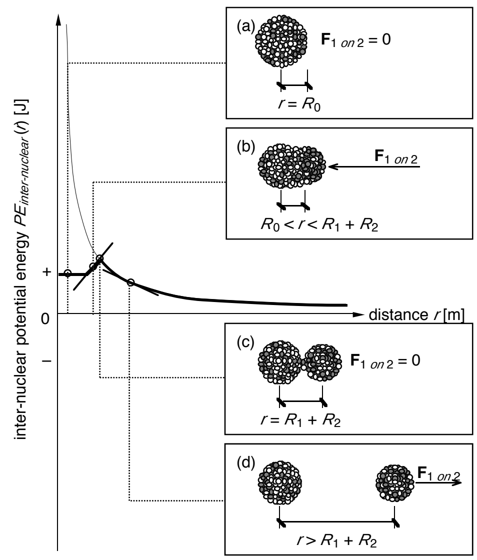
$\beta^{\pm}$ (ε) decay, γ decay } "Fermi-gas model" ("box model")
T. A. Moore, *Six Ideas of Physics, Unit Q*

> "Different model approaches try to accentuate various aspects of nuclear structure in a simple and schematic way. No single model, as yet, is detailed enough to encompass *all* aspects of the nucleus..."
> —K. Heyde,
> *Basic Ideas and Concepts in Nuclear Physics,* 1994

• Construction of internuclear *PE* graphs
General features



Internuclear *PE* graphs
- Characterizes nuclear-nuclear interactions
- *PE* gradient (slope) = force
  - Electric repulsion when separated
  - Combination of strong nuclear attraction and electric repulsion when touching (net force attractive)

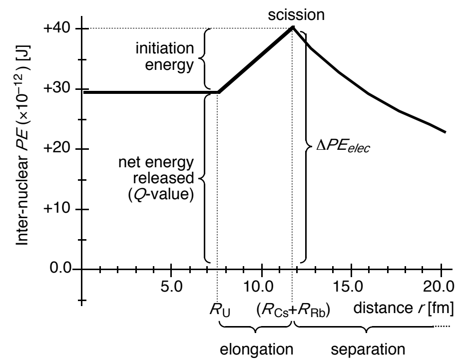


• Construction of internuclear *PE* graphs
Exothermic fission example

Induced exothermic fission reaction:

$$n + {}^{235}_{92}U \rightarrow {}^{93}_{37}Rb + {}^{141}_{55}Cs + 2n + \left[28.8\times10^{\pm12}\ J\right].$$

Construction of internuclear PE graph:
- Radii calculated from $R \approx (1.3\ \text{fm})A^{1/3}$.
- Energy at scission point calculated from PE_{elec} of daughter nuclei
- Assume straight-line segment during elongation



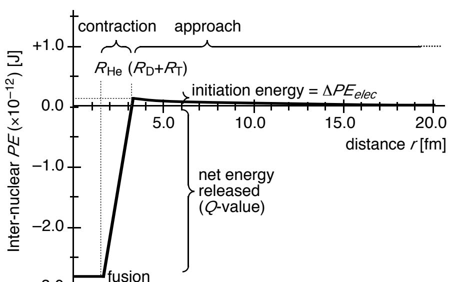
What can this graph tell us?
- *PE* gradient (slope) = net force between daughter nuclei during elongation or separation
- Required initiation energy from neutron (overestimated, if tunneling effects are not included, and graph not smoothed at scission point)

• Construction of internuclear *PE* graphs
Exothermic fusion example

Exothermic fusion reaction:

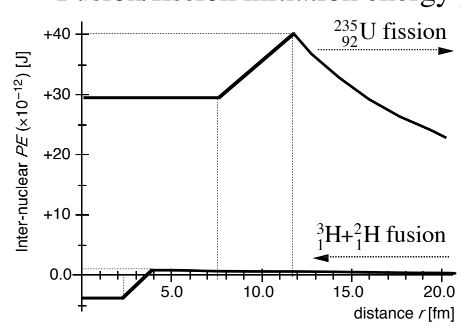
$${}^{3}_{1}H + {}^{2}_{1}H \rightarrow {}^{4}_{2}He + n + \left[2.81\times10^{\pm12}\ J\right].$$

Construction of internuclear PE graph:
- Radii calculated from $R \approx (1.2\ \text{fm})A^{1/3}$.
- Energy at fusion point calculated from PE_{elec} of D and T nuclei
- Assume straight-line segment during contraction



What can this graph tell us?
- *PE* gradient (slope) = net force between daughter nuclei during elongation or separation
- Required initiation energy (comparison with fission later)
- Relative initiation energies for various nucleosynthesis processes can be compared
- In general:
  - exothermic fusion = endothermic fission
  - exothermic fission = endothermic fusion

• Analysis of nuclear processes
Fusion/fission initiation energy paradox



What can these graphs tell us?
- Both graphs constructed in the same manner
- Appreciation of relative distance and energy scales involved

Common question asked by students:
- Initiation energy for tritium-deuterium fusion is two orders of magnitude smaller than initiation energy for induced U-238 fission!
- Yet why is fusion so much more difficult to initiate than fission?

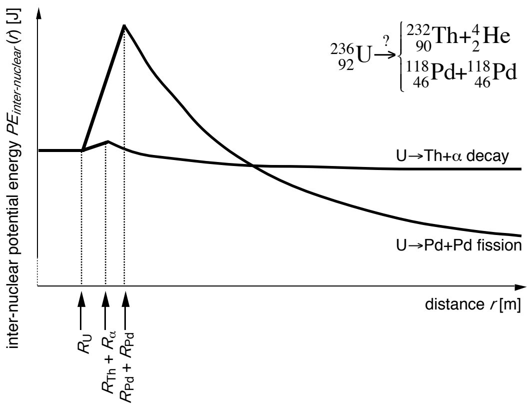
Fusion—two nuclei must come into contact
- Both nuclei repel each other
- Both nuclei move independent of each other in plasma phase

Induced fission—neutron penetrates U-238 nucleus
- No neutron-uranium repulsion
- Neutron will eventually hit a U-238 nucleus in a solid phase critical mass sample

• Analysis of nuclear processes
α decay versus "symmetric fission" paradox

Common question asked by students:
- α decay can be thought of as "asymmetric" fission, where less net energy released than fission into symmetric daughter nuclei
- Yet why is α decay observed rather than energetically favorable symmetric fission?

Compare internuclear PE graphs of both processes:
- Symmetric fission: more exothermic, but higher initiation energy (*KE* from external neutron required)
- α decay: less exothermic, but much lower initiation energy ("self-initiating" with tunneling)



• Conclusions
So what can internuclear *PE* graphs do?

Visualization of relative distance and energy scales involved in different nuclear processes

- Uses already available information:
  - Nuclear radius, calculated from nucleon number
  - *Q*-values, calculated from mass decrements
- Emphasizes reversibility of exo/endo- thermic fission/fusion processes
- Quantifies net internuclear forces during elongation/contraction phases of fission/fusion
- Graphical comparison of initiation energies for:
  - Induced versus spontaneous fission
  - Stellar nucleosynthesis processes
  - α decay versus "symmetric" fission
- More complex effects can be introduced and incorporated (such as tunneling, smoother increase in internuclear *PE* due to increase in surface area during elongation)